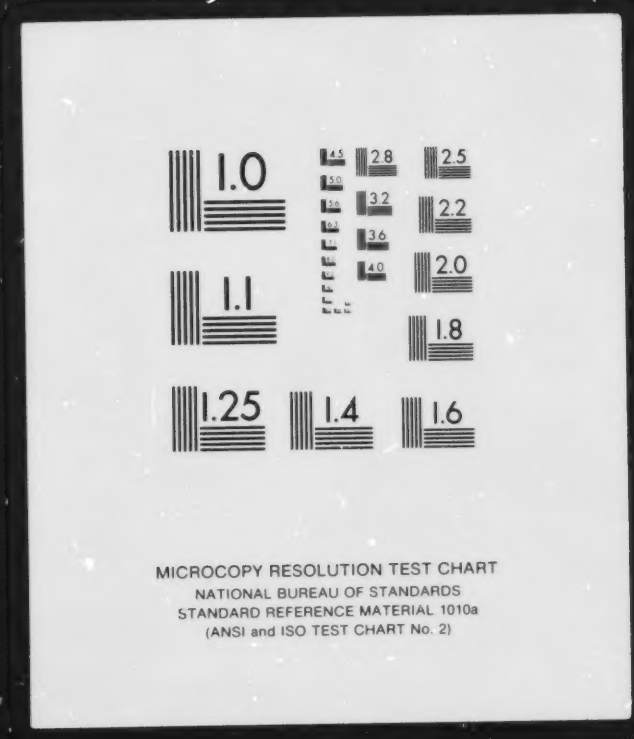
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ABSTRACT

Because of the general recognition of the importance of mathematics ability, and the close relationship between mathematics ability and spatial ability, eight studies were undertaken to discover and describe aspects of spatial competence in children. The range of abilities tapped stretched from very early precursor skills with Euclidean space to those requiring sophisticated manipulation of internal spatial images. The studies focused on two factors: spatial orientation and spatial visualization. Children from ages three to twelve years were included in the studies. Sex differences were studied across the sampling of skills and age ranges. In all studies, age was found to be a statistically significant factor. The major developmental trend across studies, indexed by the age factor, appeared to be a fairly linear quantitative growth pattern common to each. The result for the sex factor was also uniformly consistent: no significant sex differences were detected. (Author/BW)

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Acquisition of spatial concepts in relation to age and sex.

Department of Educational Psychology
University of Arizona

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I. General Information

A. Scope of the Research

It has long been recognized that the majority of scientific and technical professions requires mathematical expertise as a prerequisite to formal training. High school and college curricula invariably offer the scientifically oriented student instruction in the various facets of mathematics to insure proficiency in those crucial prerequisite skills. Mathematical "literacy," like its verbal counterpart, is an important aspect of intellectual competence. There is little doubt that, at a minimum, algebra, geometry, and calculus form a necessary basis for advanced training in scientific and technical fields. It has furthermore been noted that women are underrepresented in these technical fields (Ernest, 1976; Sells, 1976; Fox, Fennema, & Sherman, 1977), and that they demonstrate less proficiency on measures of mathematical achievement (Maccoby & Jacklin, 1974; Fox, 1977; Sherman, 1978).

Closely related to the issue of mathematical "literacy," the ability to process, transform, and manipulate quantitative information, is that of spatial "literacy," the ability to process and manipulate spatial information. If an aptitude factor is related to apparent differences in mathematics achievement, that factor may involve sophistication in the manipulation of spatial information, and here again it is generally maintained that females are less successful than males on aptitude measures designed to tap spatial abilities and spatial reasoning (Bennett, Seashore, & Wesman, 1966; Sherman, 1967; Maccoby & Jacklin, 1974; Fennema & Sherman, 1976; McGee, 1979). Moreover, formal curricula offer little to foster spatial skill acquisition (Brinkman, 1966; McGee, 1979). This presents an interesting and somewhat paradoxical state of affairs since much of the material designed for instruction in mathematical content, particularly that for younger children, relies implicitly or explicitly on spatial analogues to convey conceptual material. At the same time, however, little or no effort is expended to insure that the spatially presented information communicated the concepts to learners.

The relationship between spatial abilities and other aspects of mathematical expertise is not a clear one, and some would debate its importance (Fennema, 1977). However, spatial abilities, at the very least, have relevance for specific mathematical content areas like geometry (Bennett,

Seashore, & Wesman, 1974) and perhaps even calculus (Eisenberg & McGinty, 1977). It is interesting in this respect, that investigators have observed a close correspondence between the point at which traditional Euclidean geometry is offered and the large drop-off in the number of females in mathematics courses (Fennema, 1977). This drop-off may, however, be diminishing (Armstrong, 1979). Spatial abilities also seem to be relevant to certain male-dominated technical fields such as the design professions (e.g., drafting, architecture, and engineering) which require spatial proficiency (McGee, 1979).

There seems to be, then, a general recognition of the importance of mathematics proficiency as a relevant factor in intellectual competency. Females apparently demonstrate less proficiency. There is also a general recognition that mathematics achievement and spatial ability may be related. Females have been noted to perform less ably than males on spatial aptitude assessment. Add to these conclusions the facts that: (1) the technical professions, including those which require spatial competence, are male dominated, (2) mathematical instruction is often predicated on spatial analogues and spatial presentation, and (3) little is done in the schools to foster spatial "literacy." Neglecting instruction in spatial competence could discriminate against the less spatially-minded student, erecting a barrier that may later impede success in future mathematics proficiency. Unfortunately, a majority of those so impeded could be females.

Why does such a state of affairs exist? The answer to this question is complex involving affective, social, and cognitive factors (Fox, Fennema, & Sherman, 1977). But two possible answers can be offered. First, the sex differences literature in both the mathematics and spatial areas is equivocal and debated (see Sherman, 1978). Second, we really do not fully understand the area of spatial competence or the origins of those abilities, an issue which is also debated (Sherman, 1970; Stafford, 1972; Bock, 1973; Bock & Kolakowski, 1973; Ernest, 1976). Until we have a better understanding of spatial abilities even in a descriptive sense, we certainly can do little to foster acquisition of them.

The studies outlined in this report are addressed at the issue of discovering in a precise and systematic manner, aspects of spatial competence in children. We still require more information about what spatial abilities children possess, and in what sort of sequence they develop them, before

we can effectively foster ability development. Moreover, spatial competence is not necessarily a global ability area but may be made up of separate skills and problem-solving strategies (Lohman, 1979a). Therefore, we need not only general information about these abilities but specific information as well. An finally, the issue of sex differences needs addressing both across age levels and across skill areas. With this kind of information we will be in a better position to judge the relevance of sex differences on these measures and the nature of the abilities themselves.

B. Project Objectives

In this report, the results of eight descriptive studies will be presented. The range of abilities tapped stretches from very early precursor skills with Euclidean space to those requiring sophisticated manipulation of internal spatial images. The age range covered, children of three to twelve years of age, is similarly broad. The sex factor has been included across the sampling of skills and age ranges. Hopefully this body of data will bear directly on the general issue of spatial abilities and sex differences.

To lay an appropriate content and basis for the empirical work presented here, we will first present a general theoretical background and review of past research on these issues. Second, the studies themselves will be presented, accompanied by the particular psychological rationale that served as their impetus. And finally, a general disucssion will be presented relating back to the general integrating research goals.

II. Theoretical Perspective and Review of the Literature

A. The Developmental Approach

Our interest is in the learning of spatial and mathematical skills in children. To build the kind of systematic research undertaking proposed here, one requires a sound theoretical base, which can be drawn at least in part from developmental psychology. Mathematics knowledge and its emergence is of primary interest to those concerned with developmental issues. One obvious reason for this is the impact of Jean Piaget in stimulating research into the development of children's mathematical thinking. His writings present insightful observations into children's emerging notions of mathematics, imagery, the concepts of space, spatial relationships, and and the successive changes these concepts undergo in their evolution

(Piaget & Inhelder, 1956; Gruber & Voneche, 1978). Piaget views man as an active constructor of a knowledge system. His ontogenetic investigations reflect questions of how the system is built into a complex, coordinated mature structure which contains sophisticated ideas about space and spatial relations, as represented formally by mathematicians (Flavell, 1963; Gruber & Voneche, 1978).

Essentially, Piaget and Inhelder have broken their investigations of spatial representational phenomena into three sections: (1) studies of topological spatial relations that consider elements and characteristics of objects in isolation, (2) investigations of projective spatial relations where an object or group of objects is considered in relation to a coordinated system of perspectives, and (3) examinations of concepts in Euclidean space that emphasize the quantitative measurement of proportions and distance.

Novel tasks which differentiate the possessor of a mature conceptualization of space from individuals with less mature ones, and at the same time reveal the young child's lack of sophistication with Euclidean characteristics, were introduced by Piaget and Inhelder (1956, 1971). They cogently argue that the young child's spatial system is not organized on the basis of Euclidean properties but rather on topological ones. Furthermore, Piaget and Inhelder (1956) outline a spatial concepts developmental sequence beginning with topologically organized cognitive structures which appear first in the developing child and remain characteristic of spatial thinking until the child is approximately four years old. This stage is followed by the concurrent, interdependent emergence of projective and Euclidean spatially organized structures which derive from the earlier topological notions. The final step in the developmental sequence relates to the potential of the cognitive structures to facilitate the precise measurement of abstract objects in accordance with Euclidean characteristics. This culminates at approximately eleven years of age.

Since the appearance of Piaget and Inhelder's seminal work (1956) there have been myriad investigations into spatial awareness and representation which either provide support for Piagetian findings or which dispute the work. For example, Laurendeau and Pinard (1970) replicated the original studies with greater precision and have essentially corroborated Piaget and Inhelder's findings while speculating that a fully elaborated understanding

of spatial relationships is not reached until approximately age fourteen and may actually derive from formal thought. Results consistent with this hypothesis were found by Cox (1978) who agrees that mastery of at least one aspect of projective and Euclidean space arises quite late in development. Dodwell (1963), in his assessment of the progression from topological through Euclidean spatial concepts, confirmed the types of thinking and problem solving outlined by Piaget and Inhelder (1956). However, he found it impossible to assign any child to a particular stage of development and failed to discover any clear-cut progression from one type of spatial thinking to another. On the other hand, in examinations of topological space, Kapadia (1974) argues that Piaget and Inhelder's description of the characteristics of topological relations is mathematically incorrect. Martin (1976a) employs empirical evidence to dispute the notion that projective and Euclidean concepts derive from the original topological concepts.

Still another area of spatial concepts research has focused on the spatial coordination of perspectives represented by the classic Piagetian three mountain problem. Presented with a model of a three-dimensional mountain landscape, children are requested to indicate a view in correspondence with the correct perspective of positions other than their own. Essentially, three levels of understanding can be distinguished in task accomplishment. Young children (4-5 years of age) cannot discriminate and coordinate the relationship between the "viewed" and the "point of view," and respond by choosing the alternative consistent with their own perspective, the egocentric response. Transitional children (age 6) seem to recognize the uniqueness of differing perspectives, but cannot determine the particulars of another's view; they make errors of an essentially random variety. Genuine comprehension is not evidenced until about age nine and even later as task difficulty increases (Piaget & Inhelder, 1956).

The three mountain problem is, however, a difficult task particularly for young children. Only rarely do children of preschool age experience success, although such results have been found (Borke, 1975; Horan & Rosser, 1979). When one considers the cognitive requirements of the task, young children's lack of success is not surprising. First, the subject must recognize and retain the spatial relationships present within the original display. Second, those identified relationships must be held constant, or

conserved, as the display is reoriented in space. If the child cannot separate these two relationships, i.e., relations among the objects and relation of the objects to the spatial field, failure will result. If the original display contains Euclidean relationships, the task becomes one of preserving Euclidean features following the motion of rotation, a rigid transformation. It is, therefore, not surprising that young children experience difficulty with this task; inferring the precise cause of that difficulty, however, is more ambiguous.

Many of the visual perspective coordination studies have included methodological variations capable of affecting spatial performance in perspective coordination, or have provided a more detailed substage delineation to elucidate the perspective construction of children at different ages (Masangkay, McCluskey, McIntyre, Sims-Knight, Vaughn, & Flavell, 1974; Flavell, 1976; Lempers, Flavell, & Flavell, 1977). From this body of work, both sequencing and methodological issues have been somewhat clarified but almost exclusively with respect to this particular aspect of spatial problem solving. Thus, only a very limited empirical view of general spatial representational abilities has been well researched.

In a broader sense, Piaget ties his specific tasks to the larger issue of spatial representation in general, perspective coordination being only one aspect of spatial representation. The Child's Conception of Space (Piaget & Inhelder, 1956) predates the work on spatial imagery (Piaget & Inhelder, 1971), but they would appear to be theoretically congruent and to deal with many of the same issues (Gruber & Voneche, 1978). Spatial imagery is viewed by many as the mechanism for success on spatial tasks (Shepard & Metzler, 1971), including rotational tasks (Huttenlocher & Presson, 1973). Hence, the Piagetian approach to this set of abilities appears to provide at least a partial theoretical framework on which to base research into spatial abilities.

Important to the goals of this proposal is that sex differences in spatial abilities are not regarded as a myth (Maccoby, 1966; Maccoby & Jacklin, 1974; Schratz, 1978). The research evidence on which sex differences in spatial visualizing is based, has primarily employed non-Piagetian spatial type tests (e.g., Embedded Figures Test, Rod and Frame Test, Body Adjustment Test, and Block Design Test). However, high correlations have been

found between these types of measures and Piagetian perspective tasks (Okonji & Olagbaiye, 1975). Direct investigations of sex differences on Piagetian type spatial visualization tasks are rarer perhaps because developmentalists more characteristically examine age-related rather than sex-related characteristics. Some of these studies omit sex as a variable, study children younger than the age at which sex differences would be expected, or find no differences when the factor is included (Rubin, Attewall, Tierney, & Tumolo, 1973; Borke, 1975; Liben, 1978a). Others, however, have found sex differences on Piagetian type spatial tasks (Coie & Dorval, 1973; Liben, 1974; Bielby & Papalia, 1975; Thomas & Jamison, 1975). Those sex differences, when found, frequently favor males (Tuddenham, 1970).

One study examined grade levels two through seven in an attempt to clarify the sex difference issue and to illuminate the relationship between spatial abilities and math achievement (Guay & McDaniel, 1977). In that study, high level spatial visualizing was measured with tests based on the Piagetian notions of the coordination of perspectives and ideas of surface development. These investigators found a positive relationship between math achievement and spatial visualization skills even at the elementary school level, and moreover, a significant effect for sex was found across grade level on the spatial visualization tests, with the difference consistently favoring boys.

This suggests that sex differences may arise both on traditional tests of spatial relations, and Piagetian-derived tasks as well. Moreover, these differences may occur prior to adolescence. If we wish to understand why the sex discrepancy exists at the later point in development, perhaps we need to focus on the early precursors of that discrepancy. Developmental theory gives us focus and direction to examine those skills prerequisite to and precursors of mature geometric and spatial reasoning.

B. Individual Differences and Spatial Abilities

The cognitive developmental sequence of Piaget and Inhelder (1956) is not the only available organizing device for spatial abilities. Researchers concerned with the nature of intelligence in psychometric terms have been examining individual differences in spatial abilities for decades (see Lohman, 1979a and McGee, 1979 for a review). Detected differences are purported to be related to sex, and a number of authors have reviewed

the existing literature and concluded that males are superior to females in spatial ability (Fruchter, 1954; Anastasi, 1958; Smith, 1964; Tyler, 1965; Maccoby, 1966; Maccoby & Jacklin, 1974).

Recently, however, the general acceptance of sex differences in spatial abilities has come under attack from several directions. First, it has been argued that spatial aptitude is not a well defined construct in terms of psychological process variables (Lohman, 1979a, 1979b). Second, the literature on which conclusions about sex differences are based, is actually equivocal. The age of the subjects and the type of tasks used for assessment are important variables influencing whether or not differences are detected (Fairweather, 1977). Most sex differences arise with post adolescence subjects, and the evidence for differences in preadolescents is particularly sparse (Fairweather, 1977; Sherman, 1977; McGee, 1979). The question of preadolescent sex differences is extremely relevant to theoretical attempts to explain observed differences. A lack of differential spatial performances in younger children is seen by some as evidence for an environmental etiology (Sherman, 1977; McGee, 1979).

However, even describing the precise nature of spatial ability is difficult. In the past, specification of the domain has been pursued with factor analytic studies of group intelligence test batteries (Lohman, 1979a, 1979b; McGee, 1979). Results from numerous analyses by a variety of authors have yielded much confusion (McGee, 1979). There have been, however, several convergent trends identified. In an extensive review and reanalysis of the factor analytic literature, Lohman (1979a) summarizes the current state of knowledge.

Three major spatial factors have been identified. Lohman (1979a) calls the first factor spatial relations. It is defined by such group tests as Cards, Flags, & Figures (Thurstone, 1938) as well as the experimental tasks of Shepard and his associates (Shepard & Metzler, 1971). The common element in these tests is alleged to be the mental rotation of a visual stimulus. Latency (speed of performance) is the typical dependent measure utilized. A second factor identified by Lohman (1979a) is termed spatial orientation. This factor involves the ability to anticipate how a stimulus array appears from various perspectives. Piagetian tests, involving the coordination of perspectives, are an example of this factor. Spatial orientation is also present in several established group test batteries (Thurstone,

1938; French, 1951). The third factor, visualization, has been defined by tests such as the WAIS block design, copying, and surface development (Lohman, 1979a). This factor represents performance involving the reproduction and/or combination of spatial information at the internal level. Typical experimental tasks include the reproduction of visual displays or the mental unfolding of geometric shapes (Brainerd & Heuvel, 1974).

Much of the evidence that gave rise to the issue of sex differences in spatial ability emerged from the standardization of group intelligence tests. The bulk of the cases involved the factor labeled visualization, and all employed subjects aged 16 or older. For example, Mattazaro (1972) reported a significant difference in favor of males in the 1955 standardization sample for the Block Design subtest of the WAIS. Bennett, Seashore, and Wesman (1966) report a male superiority on the space battery of the Differential Aptitude test which consists primarily of visualization type tasks. Yen (1975) reports a male advantage on the Primary Mental Abilities Spatial Relations subtest, the Form Board Test (DAT) and paper folding (DAT) all of which load heavily on the visualization factor.

There are, however, many studies which report no sex differences (see Fairweather, 1977; Sherman, 1978, for reviews) on visualization tasks. Additionally, the importance of observed differences, in terms of the small percentage of variance accounted for, has also been questioned (Sherman, 1978).

Some evidence exists for post-adolescent differences on the spatial relations factor. Yen (1975) reports an advantage for males on a mental rotation test as does McGee (1979). Metzler and Shepard (1974) and Kail, Carter, and Pellegrino (1979) both report steeper slopes for females than males when latency is plotted as a function of degrees of mental rotation. It should be noted that only two of these studies (Yen, 1975; McGee, 1979) deal with the question of correct performance, or what has been referred to as the power issue (Lohman, 1979a). Kail, Carter, and Pellegrino (1979) and Metzler and Shepard (1974) report latency differences or speed of performance may in fact be unrelated to power of performance.

Turning to preadolescent sex differences, one is confronted with a paucity of evidence to substantiate a claim of male superiority. Fairweather (1977) reviews several studies, mostly relating to the visualization

factor, which reveal an absence of sex effects. The little evicence that does exist for preadolescent sex differences is dispersed across the various factors. Coie and Dorval (1974) report a male advantage on a perspective taking task. Liben (1974) finds a similar result with a Piagetian test of horizontality (i.e., the water level task). Guay and McDaniel (1977) report an advantage for elementary school males on high and low level spatial visualization tasks.

In summary, there appears to be some evidence for a post-adolescent male advantage in visualization and spatial relations type tests. Contradictory evidence does exist, however, and the importance of the observed differences have been questioned. No clear picture of preadolescent spatial abilities emerged. The evidence is ambiguous and observed differences have been widely dispersed across the various factors.

The rationale underlying the use of factor analysis to understand individual and group differences is an assumption that the shared variance accounted for by a factor represents some sort of psychologically meaningful entity. If males outperform females on tests which load heavily on the visualization factor, they are said to be better at the psychological abilities indexed by that factor. Therefore, the factor is treated as an external indication of certain psychological processes common across subjects. According to this conception, individuals solving visualization tasks, for instance, should all be utilizing related cognitive processes.

Lohman (1979a, 1979b) has cast doubt on this assumption. In his reanalysis of the major American factor analytic studies (1979a) evidence was uncovered which raises questions concerning how isomorphic factors are to specific psychological processes:

1. Different subjects report different strategies for solving the same items (i.e., not all subjects solve items, related to a particular factor, in the same way). Some report spatial strategies, others report using verbal-analytic means.

2. Subjects report solving items within a test by different means, depending on the complexity of the item. The amount of verbal analytic strategies used increases as item complexity increases.

3. Factoring within subjects using a particular strategy yields different factor structures than factoring across strategies. The same is true

for age and ability level. The factor structure may represent an artifact of the group chosen rather than any description of the structure of individual's abilities.

4. Spatial relations tests are usually speeded, while visualization and spatial orientation tests are not; Lohman's (1979b) work seems to indicate that speed of performance is independent of power. The spatial relations factor may be just an artifact of this independence.

The tests used in factor analytic studies represent problem-solving situations of various complexities. Just what types of operations and what item complexities prove differentially difficult is not identified by a factor analytic approach. Factors represent different concatenations of variables, some of which may not even be cognitive, across subpopulations and situations.

C. Conclusions

If one wishes to understand how young children progress in their acquisition of spatial concepts and competencies neither the correlational nor the developmental perspectives provide a ready solution. For example, the Piagetian perspective tasks are probably not the appropriate starting place, although the general theoretical formulation is enlightening. The tasks are complex, requiring both knowledge of the spatial relationships among objects, and knowledge of the relationship between the objects and space as a container of those objects. In the three mountain-type problem those elements are confounded; failure on the overall task does not reveal which component is most problematic. Despite replications of the original findings Laurendeau & Pinard, 1970), other studies in which the task is simplified or changed have led to different findings and conclusions (Shantz & Watson, 1971; Coie, Costanzo, & Farnill, 1973; Borke, 1975; Fehr, 1978; Horan & Rosser, 1979). When the tasks are restructured in congruence with the response capabilities of the younger child, subjects evidence greater understanding (Brodinsky, Jackson, & Overton, 1972; Fishbein, Lewis, & Keiffer, 1972; McIntyre, Vaughn, & Flavell, 1973) leading to the allegation that the original tasks are noisy and insensitive to the abilities they are designed to measure (Flavell, 1976) and that the findings themselves are task specific. A less confounded task which allows separation of specific features may be more revealing.

Piagetian tasks designed to tap earlier skills, however, also present problems (Piaget & Inhelder, 1956). Essentially, these tasks try to determine whether the young child conserves topological characteristics or Euclidean ones; the original conclusions supported a topological system of space for the young child (Piaget & Inhelder, 1956). More recent evidence, however, casts considerable doubt on that conclusion both on mathematical (Kapadia, 1974) and on empirical grounds (Martin, 1976a, 1976b; Geeslin, 1979). It may well be that Euclidean and topological characteristics cannot be so easily separated and remain confounded in these tasks (Martin, 1976c).

In order to confront this issue, we chose to develop new tasks deriving from a Piagetian perspective but also incorporating the factor analytic work as well. Since our target population was young, tests that were manageable by very young children were necessary.

To conduct this series of investigations in a systematic way, we drew on a number of conceptual and methodological areas which have bearing on the acquisition of mathematical skills in general and spatial skills in particular, including work conducted by those interested in developmental acquisition (Piaget & Inhelder, 1956; Berman, Cunningham, & Harkulich, 1974; Berman, 1976; Berman & Cunningham, 1977), those concerned with geometry specifically (Martin, 1976a, 1976b, 1976c; Lesh, 1978), those concerned with spatial imagery (Piaget & Inhelder, 1956, 1971; Marmor, 1975; Kosslyn, 1978), as well as the factor analytic work (Lohman, 1979a).

III. Examination of Precursor Abilities

A. Statement of the General Problem

Typically, when Piagetian-style problems are presented to children younger than the concrete operational stage, failure results. Unfortunately children's failures of this sort inform us more about what children do not know than do they reveal the capabilities youngsters possess. Perhaps this explains why the preoperational period is one characterized by inability rather than a specification of competencies (Flavell, 1963; Gelman, 1978). Identification of very young children's skills in the spatial representation area, what they do know, is the issue to which the first four studies in this series are addressed. The attempt was made to examine Euclidean concepts in young children prior to the expected development of the full

structural system associated with a mature understanding of space.

Examination of spatial competencies in very young children poses a number of problems both conceptual and methodological. While the aim may be to expose precursors of a mature concept, eliciting those precursors is far from easy. Essentially one attempts a downward extension of prototype problems designed to detect the earliest evidence of emerging competencies. However, downward extensions examined in this fashion are more frequently found in perspective-taking tasks than they are in examinations of Euclidean principles.

The classic Piagetian-style tasks for Euclidean space include (1) conservation of length which taps understanding of the invariance of quantitative measurement, and (2) the water-level problems which examine the child's ability to display comprehension of a conceptual coordinate system. Past research suggests that very young children would perform poorly on the first type of task (Dean, 1976, 1979) and even more poorly on the second variety (Abravanel & Gingold, 1977; DeLisi, DeLisi, & Youniss, 1978). In fact, the water level task has been described as very difficult even for subjects in late childhood and adolescence (Liben, 1978a). Since young children will invariably fail to display competence on these tasks, they do not offer much promise for assessing the earliest emergence of related skills.

Other work has focused on a limited range of precursor skill development such as the orientation of a straight line (Berman, 1976; Berman & Cunningham, 1977). However, between the orientation tasks, which tap a single Euclidean feature, and the Piagetian tasks, which require a mature understanding and coordination of a number of the features, exists a sequence of conceptual complexity about which little is known. Identifying those specific skill components associated with spatial accomplishments, which emerge prior to full understanding, would appear to require an intermediate series of tasks.

One means for examining the intermediate skill range is to develop a variety of spatial problems, which gradually increase the number of Euclidean features considered and which can be hierarchically arranged on that basis. This was the route chosen for the present studies. Moreover, we chose to eliminate particular Euclidean features which previous research had indicated would be problematic for young children, that is, conserving distance and precise measurements and performing in the absence of a

facilitative reference frame. These particular features may be the ones which make the classic Piagetian tasks so difficult for young children. By eliminating the problematic features, one can then look for the emergence of other aspects of Euclidean knowledge such as (1) orientation of a spatial array in a plane, (2) coordination of horizontal and vertical lines, and (3) preservation of angularity. Detection of these latter concepts, while not sufficient to infer a fully developed Euclidean spatial system, could serve as indicators of the possession of limited Euclidean understanding which will later be organized into a mature, coordinated conceptualization.

A second task characteristic crucial to the assessment of the young child's competence is the manner in which subjects must indicate a response. Developmental limitations of the young child can make it difficult to separate conceptual failures from psychomotor and language failures. Alteration in response demands may reveal differential ability patterns, as in fact has been the case with perspective-taking spatial tasks (Shantz & Watson, 1970, 1971; Borke, 1975; Horan & Rosser, 1979).

A logical response option for assessing the child's knowledge of Euclidean concepts is a drawing measure. However, though drawing has been utilized as an assessment procedure (Piaget & Inhelder, 1956), it could prove very difficult for the young child. Researchers working with older children have concluded that drawing may have limited value as a dependent measure of internal representations (Kosslyn, Heldmeyer, & Locklearn, 1977).

One additional issue that needs to be addressed when assessing the competencies of young children is the separation of perceptual and conceptual competence. While, in all probability, perceptual and conceptual knowledge of space are related (Abravanel & Gingold, 1977) the two are not synonomous. Therefore, the manner in which the child performs when operating on the perceptual level with perceptual information may differ from performance based on memory where the child operates from an internal representation of that perceptual information. To pinpoint specifics of the ontogenetic sequence, separation of the two levels of functioning seems advisable.

To address the examination of early Euclidean competencies in the first set of studies, task difficulty, response mode, and performance based on perception or internal representation were included as experimental variables. A series of sequential tasks increasing in difficulty were designed

for the purpose of eliciting early comprehension of a number of Euclidean features. In four separate studies, dimensionality of materials, type of response demand, and performance condition were varied with three- through five-year-old children serving as subjects. It was anticipated that through this careful and systematic approach, it would be possible to identify if and when very young children could display some knowledge of Euclidean principles. While performances of this kind would developmentally predate full, mature understanding, the information should illuminate specifics of the ontogenetic attainment of skill acquisition.

B. Study 1

1. Research objective. The first study undertaken during the project's activities was an attempt to: (1) tap the lowest level of Euclidean skills, (2) validate task prototypes for use in future studies, (3) and assess the precision with which an open-ended response device could be employed with preschool children. We wished to discover the precise competencies which the subject population could display and had few preconceptions of what those skills might be. In short, this was designed as an open-ended power test of low-level visualization spatial abilities.

In order to meet the research objectives specified above, we required simple tasks with straightforward materials. Therefore, two-dimensional stimulus designs using a minimum of plain materials were selected. Since the stimulus designs would serve across several studies, a large pool of items (approximately 50) were constructed, piloted, and assessed. Finally, since an open-ended power test was desired, a construction response task was attempted.

2. Method

a. Subjects. Sixty subjects ranging in age from 3 through 5 years served as participants in the study. The children were divided into three age groups: (1) three year olds ($\overline{X}_{age}$ = 3 years 6 months, (2) four year olds ($\overline{X}_{age}$ = 4 years 7 months, and (3) five year olds ($\overline{X}_{age}$ = 5 years 4 months). Equal numbers of males and females were selected from three private preschools serving middle to upper middle class families.

b. Materials. In order to develop a set of tasks which would tap a sufficient range of precursor concepts, a seven-step difficulty hierarchy was developed. Each step differed in the number of Euclidean features that

would be included in the item type. At the first level, only one feature was included, the orientation of a straight line on a plane. At the seventh level, seven features were depicted. Each skill level, a sample item, and the Euclidean features examined with the item type are summarized in Table 1.

The seven-step hierarchy served as the basis for generating stimulus items, 28 of which were selected for the final assessment device. In this manner, four parallel items assessed each step in the general hierarchy. Each stimulus display was mounted on either an 8"x8" plain white card or a 12"x12" one depending on the complexity. Displays were constructed of colored paper cutouts including (1) 1"x1" red and blue squares, (2) 5"x1" red and blue strips, (3) 4"x1" red and blue strips, (4) 10"x1" red and blue strips and (5) 1" diameter yellow circles. The cutouts were then attached to the white frames in the 28 designs. Children were to indicate their responses by constructing a duplicate display on a same-size, plain, white card using similar paper cutouts.

c. Scoring. The responses made by children were recorded in full by a response coder who drew the outcome of performances. The completed protocols were later scored by other individuals. This procedure provided a complete record, errors included, of the child's actual constructions, and did not require correctness judgments on the part of the coder. Coders were trained until the interjudgment reliability between them reached .99 .

Three types of scores were computed for each subject. Two scores were associated with correct responses and one score summarized errors. The first correct-response scoring method produced a quantitative score. Each Euclidean feature depicted in a stimulus item could receive a score of one point if the child preserved it in his/her match construction. In this way item values could vary, the lowest being worth one point (items 1-4) and the highest earning seven points (items 25-28). A child, therefore, could receive a partial score for an item depending on the actual number of Euclidean features preserved in the match.

The second scoring system for correct responses was the more usual right/wrong judgment based on the preservation of the entire display. A totally correct match received one point; anything less than totally correct was assessed as 0.

Table 1

Hierarchy Level, Item Type, and Euclidean Feature Assessed

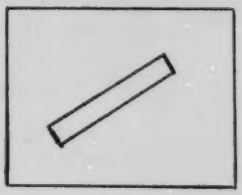
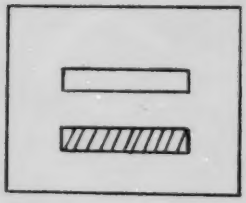
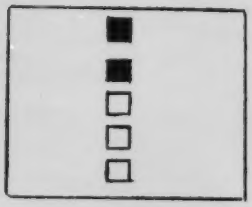
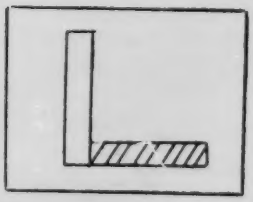
Level	Item	Feature Preserved
1		a. Orientation within the plane
2*		a. Orientation on correct side of provided rod. b. Parallelness
3		a. Orientation b. Straightness c. Order of colors
4		a. Orientation b. Point of Intersect c. Congruence of 90^{o} angle d. Connectedness of rods

Table 1--Contined

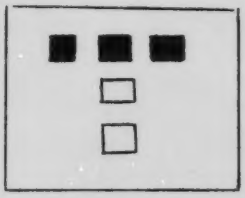
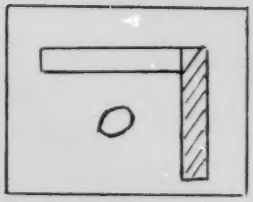
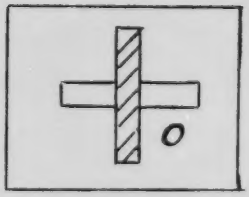
Level	Item	Features Preserved
5		a. Orientation b. Straightness c. Order of colors d. Point of interest e. Congruence of 90° angle
6		a. Orientation b. Point of intersect c. Congruence of 90° angle d. Connectedness of rods e. Location of object within axis f. Object placed distinct from axis
7		a. Orientation b. Point of intersect c. Congruence of angles d. Connectedness of rods

Table 1--Continued

Level	Features Preserved
7 Continued	e. Object placement in correct left-right orientation f. Object placement in correct quadrant orientation g. Object placed distinct from axis

*Child need only place one rod, the other is provided on the response device.

Error scores were also computed. Each error produced by children was categorized as either one which depicted a rigid transformation (rotation or reflection) of the original display, or as one which violated at least one of the other Euclidean features under consideration. Error scores were of a proportion form for each individual computed as

$$p = \frac{\text{Number of reflections and rotations}}{\text{total errors}}$$

Hence the resulting number indicated the proportion of errors of a rigid transformation type.

With these scoring systems each child received three scores: (1) a quantitative score, with a possible value range of 0-112, (2) a score for number of fully correct responses, possible range 0-28, and (3) a proportion error score, possible range 0-1.1 . The reliabilities of the quantitative scores computed with the Spearman-Brown split-half method indicated an r=.95 . Interjudger reliabilities on the other systems were similarly .95 .

d. Procedure. Each child was assessed twice by one of two female experimenters (E). An additional person was present in a coding capacity to record children's actual responses. The subject (S) was seated at a table directly opposite E and was told that he/she would be looking at some pictures. It was stressed that S needed to remember how the picture looked because S would be asked to make one just like it. Reinforcement, consisting of the presentation of small stickers, was delivered after every four items to keep the child on task. E modeled the initial item which S then duplicated to insure that subjects understood the task demands.

In the first experimental session children were assessed in the stimulus exposed condition. The item was placed flat in front of the subject for a set exposure time which varied from 6-12 seconds depending on stimulus complexity. After the specified inspection time, the white response board and necessary number of paper strips were presented to the subject who was then instructed to produce a match. All 28 items were presented in this manner in a fixed order.

In the second experimental session, conducted at least seven days after the first, children were again examined on the same 28 items by the same experimenter. This time, however, the stimulus display was removed before the child could construct a match, hence the subject performed in an unexposed condition. The order of exposure condition was held constant across

subjects with exposed always preceding unexposed. In this way any practice effect would tend to increase performance in the unexposed condition. If data still revealed a response difference favoring the exposed condition, this design factor would then serve as a conservative estimation of the difference between perceptual and conceptual performances.

3. Results

Since two different scoring methods were employed for correct responses, a preliminary analysis of both score types was conducted in order to judge the equivalence of the outcome recorded with each. Quantitative score data from the exposed condition were analyzed in a 3(age)x2(sex) analysis of variance. A significant effect was found only for the age variables ($F_{2,54} = 24.5$, $p < .01$). The same protocols scored on the right/wrong basis yielded success rate data also analyzed in a parallel analysis of variance. Again, a significant effect was found only for the age factor ($F_{2,54} = 34.5$, $p < .01$). Comparative descriptive data and analysis of variance summary tables are presented in Tables 2 and 3. Since it seemed clear that the scoring systems revealed similar outcomes with respect to success rate differences, the more sensitive quantitative score was utilized in the other complex correct response analysis.

Correct response analysis of the quantitative score data were analyzed in a 3(age)x2(sex)x2(condition) analysis of variance with repeated measures on the last factor. Significant effects were found for age ($F_{2,54} = 54.05$, $p < .001$) and for condition ($F_{1,54} = 134.27$, $p < .001$), but neither the sex main effect nor the interactions were significant. Post hoc analysis with the Newman-Keuls technique (Kirk, 1968)* indicated that three year olds did significantly less well than did four and five year olds ($p < .01$); the latter two age groups did not differ. The exposed condition proved significantly easier than the unexposed condition despite a possible practice effect. These data are summarized in Table 4.

Descriptive data were also compiled to examine the pattern of correct responses across levels of task difficulty. Using the quantitative score,

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*Newman-Keuls analyses were used in all post hoc examinations conducted in the eight reported studies.

Table 2

Summary of Preliminary Analysis of Variance: Study 1

Right/Wrong Data for the Exposed Condition

Source	df	MS	F	p
Age (A)	2	639.1	34.5	< .01
Sex (B)	1	2.8		ns
A x B	2	9.8		ns
Error	54	18.5		

Table 3

Summary of Preliminary Analysis of Variance: Study 1

Quantitative Data for the Exposed Condition

Source	df	MS	F	p
Age (A)	2	2,184.65	24.5	< .01
Sex (B)	1	14.02		ns
A x B	2	182.8		ns
Error	54	89.1		

Table 4

Summary of the Analysis of Variance: Study 1 Data

Source	df	MS	F	p
A (age)	2	5370.03	34.05	< .001
B (sex)	1	488.03	3.09	ns
C (condition)	1	5413.63	134.27	< .001
C x A	2	19.56	.49	ns
C x B	1	61.63	1.53	ns
A x B	2	395.16	2.51	ns
C x A x B	2	21.56	.53	ns
S (AB)	54	157.709		
S (AB) C	54	40.32		
Total	119			

an ascending linear function can be generated which represents the score pattern for a person performing perfectly across items. This function can then be compared to the function curve based on the actual means attained by the three age groups under the two exposure conditions. The relationships between the ideal function curves and the attained ones are depicted in Figures 1 and 2.

Pass-fail frequency data were used to examine the percent of correct responses produced by the children at each step in the hierarchy. These data, depicted in Table 5, give some indication of the actual level of proficiency exhibited by the children under the two exposure conditions. The frequency data were also used in some exploratory analysis procedures designed for the examination of categorical data (Goodman, 1978). Utilizing this type of statistical procedure, one can examine whether or not specific steps in a task hierarchy are of equal or differential difficulty, whether one step is easier than another, or whether adjacent steps are independent. These analyses suggested that steps 3 and 5 were not related to the rest but appeared to tap a different skill area. Step 1 proved significantly easier than steps 2, 4, 6, 7 which grouped together indicating a close relationships among the skills.

To further explore the possible differential difficulty of item types, the right/wrong response data were analyzed in a 3(age)x7(trials) analysis of variance with repeated measures on the second factor. Each trial was one segment of the test examined by four parallel items, hence scores on any one trial could vary from 0-4. If the steps represented on the test were in fact pulled from different domains a trials effect would be anticipated. On the other hand, absence of a trials effect could indicate a uniformity in difficulty level sampled. A significant effect was found for age ($F_{2,57} = 11.4$, $p < .001$) and for trials ($F_{6,342} = 11.4$, $p < .001$) but not for the interaction. Pairwise comparisons with the Newman-Keuls procedure indicated that: (1) Trial 1 was significantly easier than all other steps in the hierarchy; (2) there were no significant differences between Trials 2, 3, 4, 6 and 7; (3) the mean of Trial 5 was significantly lower than those for Trials 2 and 4, but not from 3, 5, and 7. All comparisons were made at the .05 level. These data would seem to support the idea of a fairly uniform domain with the initial step in the hierarchy proving somewhat easier. This is summarized in Tables 6 and 7.

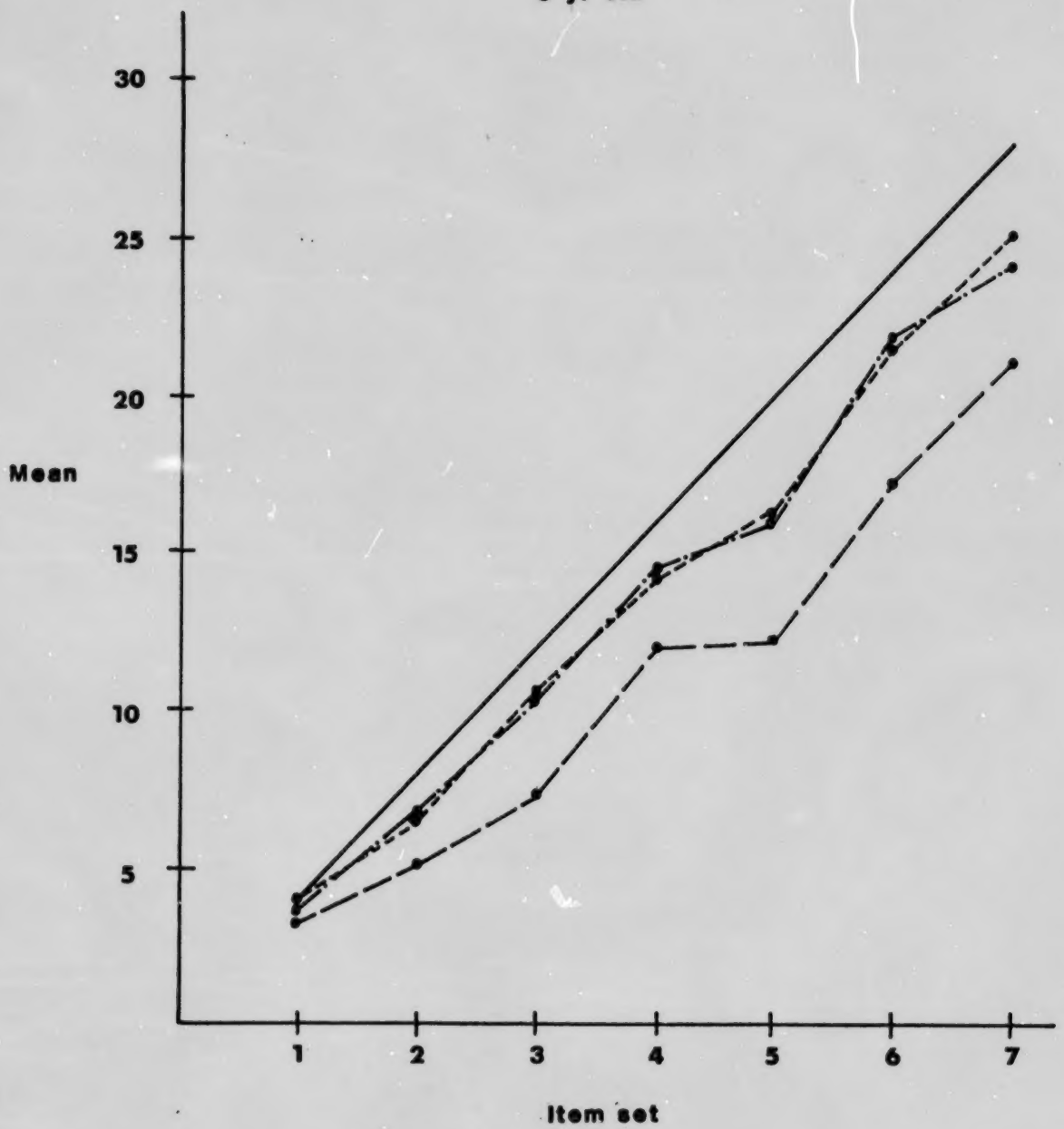


Figure 1: Means for age groups across task levels-quantitative score data. Study 1 data

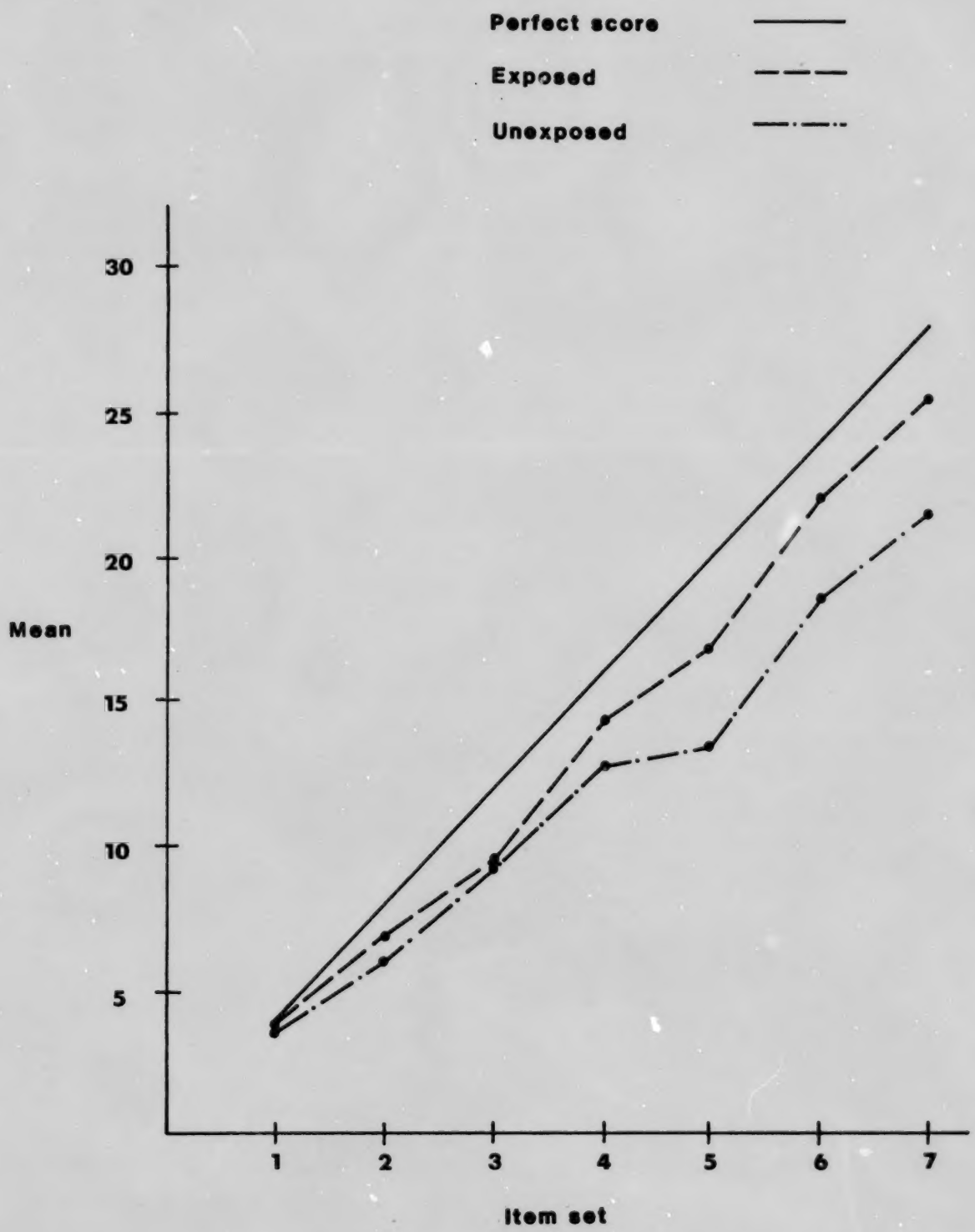


Figure 2 : Means for conditions across task levels- quantitative score data. Study 1 data

Table 5

Percent of Correct Responses as a Function of Item Level, Age, and Exposure Condition: Study 1 Data

A. Exposed Data

Item level	Age 3	4	5
1	78	98	95
2	44	80	71
3	36	73	74
4	44	85	75
5	33	61	66
6	35	80	83
7	28	75	75

B. Unexposed Data

1	76	89	99
2	33	59	63
3	34	58	71
4	16	50	43
5	5	24	30
6	9	29	29
7	9	23	26

Table 6

Descriptive Data by Hierarchy Step: Study 1 Exposed Data

			Trial						
			1	2	3	4	5	6	7
Age	3	$\bar{X}$	3.1	1.9	1.45	1.95	1.25	1.35	1.2
		SD	.96	.85	1.05	1.15	1.16	1.23	1.0
	4	$\bar{X}$	3.9	3.2	2.90	3.4	2.45	3.2	2.95
		SD	.31	.8	1.02	.68	1.43	1.0	1.09
	5	$\bar{X}$	3.8	2.85	3.0	3.0	2.65	3.3	3.1
		SD	.40	1.2	1.02	1.08	1.4	1.2	.85

Table 7

Summary of Analysis of Variance: Study 1 Exposed Data x Hierarchy Trials Responses

Source	df	MS	F	p
Between				
A (Age)	2	88.75	34.8	< .001
Error	57	2.55		
Within				
B (trials)	6	13.0	11.4	< .001
A x B	12	1.38		ns
Error	342	1.14		

Error score data were also subjected to a 3(age)x2(sex) analysis of variance for each exposure condition. For the exposed condition data, significant effects were found for both age ($F_{2,54} = 11.36$, $p < .01$) and for sex ($F_{1,54} = 6.15$, $p < .025$). For the unexposed data, no age effect was detected, but a significant sex effect was again noted ($F_{1,54} = 8.2$, $p < .01$). In the exposed condition almost 50% of errors made by males were of the rigid transformation type as compared to 22% for females. In the unexposed condition, 61% of male errors were of this type; 45% was the comparable rate for females. Tables 8 and 9 summarize the ANOVA results.

C. Study 2

1. Method

a. Subjects. Fifty-four children ranging in age from 3 to 6 years served as participants in the second study in the series. Again the children were divided into three age groups. The mean age for the three year olds was 3 years 5 months, $\bar{X}$ age four year olds = 4 years 4 months, and $\bar{X}$ age for five year olds = 5 years 2 months. Equal numbers of boys and girls were included in each age group.

b. Material. The same 28 two-dimensional items were employed in this study also. However, the response device differed. Rather than examining children's actual match constructions, in this study a selection response was chosen. For each item, children were asked to discriminate the correct option from a field of four. Each selection board for each item contained one correct option and three foils arranged in four quadrants on a 16 and 3/4" x 16 and 3/4" or 22" x 22" board, depending on item complexity. To insure complete counterbalancing of option type by position and thereby to avoid a confounding with position preference, four different response boards were constructed for each item. To facilitate errors analyses on the data, the correct option was placed in a different quadrant across each item in four response board sets. Children were then randomly assigned to selection set.

Foil types chosen for the selection response boards were based on the actual child constructions produced in the first study. For each item, high frequency errors were selected as distractors. Typically these errors included a reflection, a rotation, and one where an essential Euclidean feature other than orientation was violated. In general, the foil types

Table 8

Analysis of Variance on Error Scores: Study 1 Exposed Data

Source	df	MS	F	p
Age (A)	2	73.83	11.36	< .01
Sex (B)	1	40.0	6.15	< .025
A x B	2	.225		ns
Error	54	6.5		

Table 9

Analysis of Variance on Error Scores: Study 1 Unexposed Data

Source	df	MS	F	p
Age (A)	2	.15	3.0	ns
Sex (B)	1	.41	8.2	< .01
A x B	2	.01		ns
Error	54	.05		

were relatively close to the correct option in the hopes of requiring the children to make subtle discriminations between choices.

c. Scoring. Each item was scored as either correct or incorrect. Scores could range from 0-28. The split-half reliability computed with the Spearman-Brown formula was $r=.76$. A proportion error score was also computed in the same manner as for the first study.

d. Procedure. Each child proceeded through the experiment in nearly precisely the same manner as in the first study except that a coder was not present. Following the specified exposure time, the child was presented with the selection board, placed flat on the table, and asked to point to the option which matched the standard stimulus display. Again, the children were first assessed in the exposed condition and, at least one week later, in the unexposed.

2. Results

A preliminary analysis examined score differences on the basis of the four sets of materials used. No significant difference was detected, indicating that the sets were equivalent despite positional differences. The correct response data were analyzed in a 3(age)x2(sex)x2(condition) analysis of variance with repeated measures on the last factor. Significant effects were found for age ($F_{2,48} = 37.38$, $p < .001$) and for condition ($F_{1,48} = 12.55$, $p < .01$), with the exposed condition proving easier. There were no significant interactions, nor evidence to suggest differential performance by the sexes. Post hoc analyses indicated that five year olds outperformed four year olds ($p < .05$) who in turn significantly surpassed the three year olds ($p < .01$). Descriptive data and the summary of the ANOVA are depicted in Tables 10 and 11.

The errors analysis revealed no significant effects, trends, or patterns in terms of option choices. In fact, error rates were comparatively low with children selecting the foils at a rate predictable on the basis of chance alone.

Hierarchical analyses on categorical data (Goodman, 1978) reiterated the findings in the first study. Step 1 proved significantly easier than the others, steps 3 and 5 appeared to be independent of the rest, and the remaining steps did not differ from one another.

Table 10

Descriptive Data: Study 2

Mean Number Correct x Age

3	4	5
11.42	17.39	20.80

Mean Number Correct x Condition

Exposed	Unexposed
17.44	15.60

Table 11

Summary of the Analysis of Variance: Study 2

Source	df	MS	F	p
Age (A)	2	804.45	37.38	< .001
Sex (B)	1	50.70	2.36	ns
Condition (C)	1	92.59	12.55	< .01
C x A	2	3.78	.51	ns
C x B	1	.93	.13	ns
A x B	2	2.79	.13	ns
A x B x C	2	13.79	1.87	ns
S(AB)	48	21.52		
S(AB)C	48	7.38		
Total	107			

3. Discussion of Studies 1 and 2

From the results of the two studies conducted here, it would appear that young children of preschool age do possess some of the elements necessary for understanding Euclidean principles. While the knowledge subjects were able to display falls short of the integrated conceptual spatial system tapped by classic Piagetian tasks, the studies provide some indices of the particular Euclidean abilities possessed by preschoolers.

It should be noted, however, that two aspects of Euclidean space were held constant for the child and not assessed. Providing the child with premeasured strips rather than requiring the drawing of lines does not necessitate that the child preserve and match quantitative distances. Providing an external reference frame similarly circumvents requiring the child to construct a frame. Therefore, these results in no way negate the descriptive findings of others as to when these mature and sophisticated notions arise. Surely, these young children would not have displayed an equal level of competence had those features been allowed to vary.

However, by eliminating those two difficult dimensions it was possible to detect emerging Euclidean notions that will later come to full fruition. Examination of the proficiency rates for these young children is nonetheless impressive. A large majority of four and five year olds met with success through the entire sequence in the exposed condition; even a third of the three year olds were successful throughout the hierarchy. Moreover, all children essentially reached ceiling on the first level tapping orientation. In fact, a problem in these studies may have been a lack of sufficient ceiling. In short, these findings do not contradict Piagetian ones but instead offer a different sort of information. The results indicate what types of Euclidean spatial abilities young children can display under optimal performance conditions.

Another interesting aspect of the data is the greater proficiency demonstrated when functioning at a perceptual level rather than from memory. While in general performance was poorer in the unexposed condition, the drop in proficiency is dramatic when children must coordinate both vertical and horizontal lines (step 4). The three-year-old children exhibit very low success rates after that point in the sequence. Even the four and five year olds' correct responses dramatically decrease when coordination of the two

axes is necessary.

A third feature of the data is the linearity of the extrapolated growth curves for this conceptual domain. For the most part these curves are straight lines suggesting a quantitative rate of conceptual development.

Methodologically, these studies are also enlightening. A more sensitive measurement has been called for in the area of spatial abilities (McGee, 1979) particularly the use of a construction response to tap power aspects of functioning (Lohman, 1979b). However, requiring construction responses from young children presents a problem. One would like to elicit products revealing of internal concepts which at the same time can be reliably scored and quantified for purposes of analysis. The construction tasks employed in the first study served that function while at the same time preserving high reliability.

Moreover, the construction response mode proved to be both more reliable and more sensitive than the parallel selection response device. Incorrect choices on a selection task can only indicate error, not qualities of the error. It was clear from these data that when children erred in the selection task, they erred randomly. Random errors reveal little if anything of thinking processes, since any pointing response meets the experimental demands. The product in a construction task, however, has the potential of sensitivity to child conceptions. When reliable construction devices can be designed for young children they may provide an effective measure which ameliorates ceiling effects and produces data useful for errors analyses.

The sex differences detected in the errors analysis is also provocative. Sex differences have been found in overall success in other studies examining Euclidean knowledge, but that result is typically found with older children, adolescents and adults. While a general difference in terms of success was not detected here, an error type difference was found. Preschool girls more often than boys maintained a correct orientation while sacrificing such features as angle preservation and object location. Boys tended to disregard the general orientation while conserving the others.

It remains to be seen how proficiently young children can perform when the more difficult Euclidean characteristics are not held constant. Children may produce less accurate matches when the external frame is removed or when they must construct their own linear relationships without preset quantitative limits. Under optimal conditions, however, it is clear they

can display elements of capability and knowledge not previously revealed.

D. Study 3

1. Research Objective

The two earlier studies, using two dimensional displays and responses, had (1) revealed age differences among 3, 4, and 5 year olds; (2) indicated proficiency rates for the preschool children, and had (3) generally detected an absence of sex differences regarding overall successful performances. Whether the same patterns would be found using three dimensional materials remained open to question. There is evidence that dimensionality may be an important task variable affecting children's performance on spatial tasks (Horan & Rosser, 1979). This third study, therefore, was an attempt to extend the examination to three-dimensional materials and at the same time to directly compare the response modes examined separately in Studies 1 and 2.

2. Methods

a. Subjects. Fifty-four children ranging in age from 3 to 6 years served as participants and were again divided into three age groups. The mean age for the threes was 3 years 7 months, $\overline{X}$ age of fours = 4 years 5 months, $\overline{X}$ age of fives = five years 4 months. Subjects were drawn from the same population pool as those in the first two experiments.

b. Materials. Twenty-eight Euclidean geometric displays, identical in form and feature to the earlier two-dimensional items, were constructed from three-dimensional blocks. Twenty-four of the cardboard bases to which the blocks were attached measured 8"x8"; four of them measured 12"x12". The blocks were of five varieties: (1) one inch cubes, (2) rectangular rods measuring 1"x1"x4", (3) rectangular rods measuring 1"x1"x5", (4) 1"x1"x10" rods, and (5) 1"x1" discs. The cubes and rods were painted either blue or red; the discs were painted yellow.

The original 28 items from studies 1 and 2 were divided into two parallel sets which seemed permissible based on the high reliabilities that had been previously achieved. One set, 14 items, would serve as stimulus displays for the construction-response mode, while the remaining 14 would serve as selection items. The pool of blocks described above were used to construct the 14 selection response boards. White cardboard backgrounds measuring 16 and 3/4" x 16 and 3/4" or 22"x22" were divided into four

square fields of equal size. In each of the four fields, the correct response or one of three foils was constructed identical to the response boards employed in study 2. White cardboard bases and additional blocks were used for the construction response items, paralleling study 1. In short, materials were constructed in a duplicate fashion to the earlier ones with the exception that (1) there were half as many items of each type, and (2) the displays contained three-dimensional blocks rather than two-dimensional strips.

c. Procedure. Each subject was tested individually on two occasions at least one week apart. Two people were present during the testing: (1) an experimenter who actually administered the items, and (2) a coder who recorded the child's responses.

Administration conditions and directions duplicated those employed earlier. However, within each session two types of responses were required of the child on different items. All 14 construction items and 14 selection items were administered in separate blocks within each session; the order of type of response blocks was counterbalanced across subjects. Both assessment sessions were identical; in the first session children responded with the stimulus display exposed and in the second those displays were removed for the unexposed condition.

3. Results

Each child's protocol was scored for total number of responses that were fully correct. Since we wished to directly compare performance on the construction items with performance on the selection items, the quantitative score had to be abandoned. Therefore, for construction items, responses had to be totally correct on each Euclidean feature to be given one point.

A split-half reliability coefficient was computed for the 28-item test. It yielded an r=.88 corrected for length by the Spearman-Brown formula. Additional reliability coefficients were computed for the construction and selection response blocks, with r's of .84 and .75, respectively. The correlation between scores on the construction and selection task was found to be .60 .

The data analysis treated the assessment device as two separate 14-item tests, one with a construction response and one with a selection response,

both with a possible score range of 0-14. Means by age, sex, exposure condition, and response mode are presented in Table 12.

The data were analyzed in a 3(age)x2(sex)x2(exposure condition)x2(response mode) analysis of variance, the last two factors serving as within variables. The summary of the ANOVA is presented in Table 13. A significant main effect was observed for exposure condition ($F_{1,48} = 60.7$, $p < .001$) with the exposed condition proving easier as expected. These data are depicted in Figure 3. The main effect for age was also significant ($F_{2,48} = 23.1$, $p < .001$). Post hoc analyses revealed significant differences for the 3's vs 4's, and 3's vs 5's only (see Figure 4). The construction response proved to be significantly more difficult than the selection response ($F_{1,48} = 6.46$, $p < .05$) as depicted in Figure 5. However, a significant task x condition interaction was also detected ($F_{1,48} = 17.83$, $p < .001$) as depicted in Figure 6. Post hoc analyses revealed that the relative difficulty of the construction task was limited to the unexposed condition. No other significant effects were detected.

Performance on the test as a whole (i.e., number correct out of 28 items) was also compiled and presented in Table 14. Errors analyses parallel those employed in the first two studies. Hierarchical analyses reiterated previous findings.

4. Discussion

Overall, the results indicate that preschool children are fairly competent at reproducing a static three-dimensional visual display of Euclidean geometric patterns. The performance of the three year olds is surprising. Piaget and Inhelder's (1956) work suggests that such young children would have difficulty reproducing Euclidean features and yet the subjects in this study did demonstrate some degree of competence. Three year olds were able to succeed, on the average, in 10 out of 28 items with the display absent and almost 14 of 28 items with the display present. Both of these performances were well above chance levels.

Performance on the construction task is particularly impressive. While a 25% chance of guessing the correct response exists for the selection task, the probability of a correct guess on the construction task must be extremely low considering the number of possible ways the items could be constructed. To give an illustrative example, even as simple an item as a

Table 12

Summary of Descriptive Findings: Study 3 Data

		3				4				5					
		M		F		M		F		M		F			
		Con	Sel	Con	Sel	Con	Sel	Con	Sel	Con	Sel	Con	Sel	Cond	Task
EXPOSED		6.33	6.00	6.56	8.22	9.78	9.78	10.78	10.0	11.11	10.33	11.22	11.2	9.28	7.82
UNEXPOSED		3.33	5.89	5.00	6.33	6.78	8.89	6.56	8.44	8.79	9.78	7.56	7.56	Unexp 7.16	Sel 8.62
Task x age	Con 5.31	Sel 6.61		Con 8.47	Sel 9.28		Con 9.67	Sel 9.97							
Age x sex	M 5.39	F 6.53		M 8.81	F 8.94		M 10.00	F 9.64							
Age x con	Exp 3 6.78	Unexp 3 5.14		Exp 4	Unexp 4 10.08		Exp 5 7.67	Unexp 5 8.67							
Age	5.96			8.88			9.82								

Males = 8.06

Females = 8.37

Key

Con=construction exp=exposed condition

Sel=selection Unexp=unexposed condition

M=male

F=female

Table 13

Summary of the Analysis of Variance: Study 3 Data

Source	df	MS	F	p
C (Condition)	1	242.78	60.7	< .001
T (response mode)	1	35.04	6.46	< .05
A (age)	2	291.68	23.1	< .001
G (sex)	1	5.04	.40	ns
CT	1	38.34	17.83	< .001
CA	2	3.19	.79	ns
CG	1	12.04	3.01	ns
TA	2	4.50	.83	ns
TG	1	.12	.02	ns
AG	2	10.5	.83	ns
CTA	2	1.68	.78	ns
CTG	1	4.45	2.07	ns
CAG	2	2.72	.68	ns
TAG	2	1.18	.22	ns
CTAG	2	4.13	1.92	ns
S(AG)	48	12.63		
S(AG)C	48	4.0		
S(AG)T	48	5.42		
S(AG)CT	48	2.15		
Total	215			

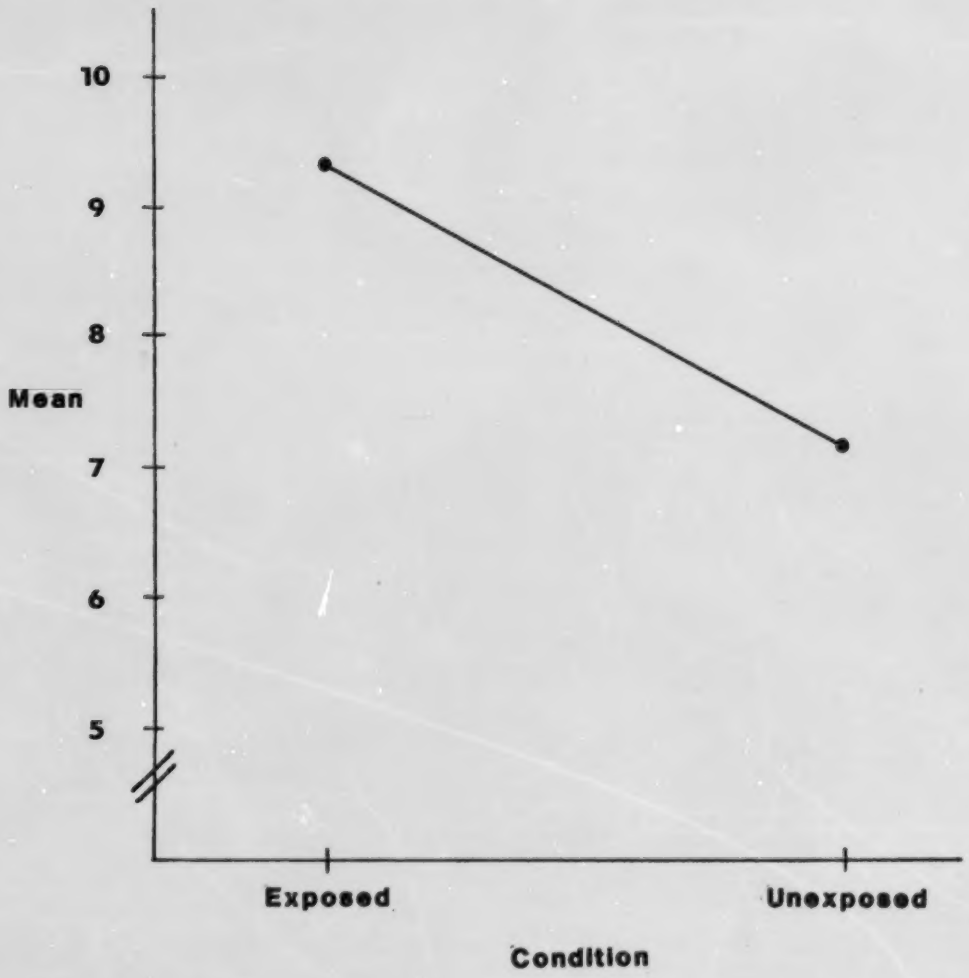


Figure 3: Mean number correct as a function of condition. Study 3 data

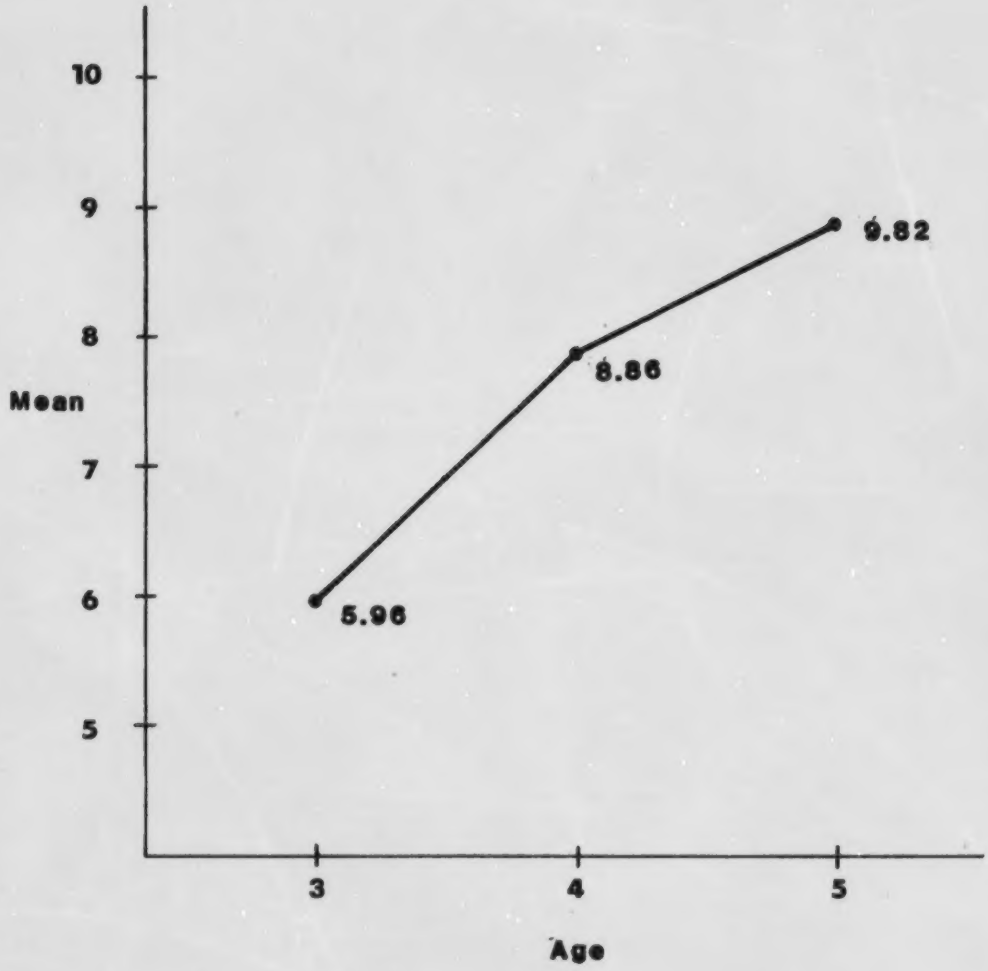


Figure 4: Mean number correct as a function of age. Study 3 data

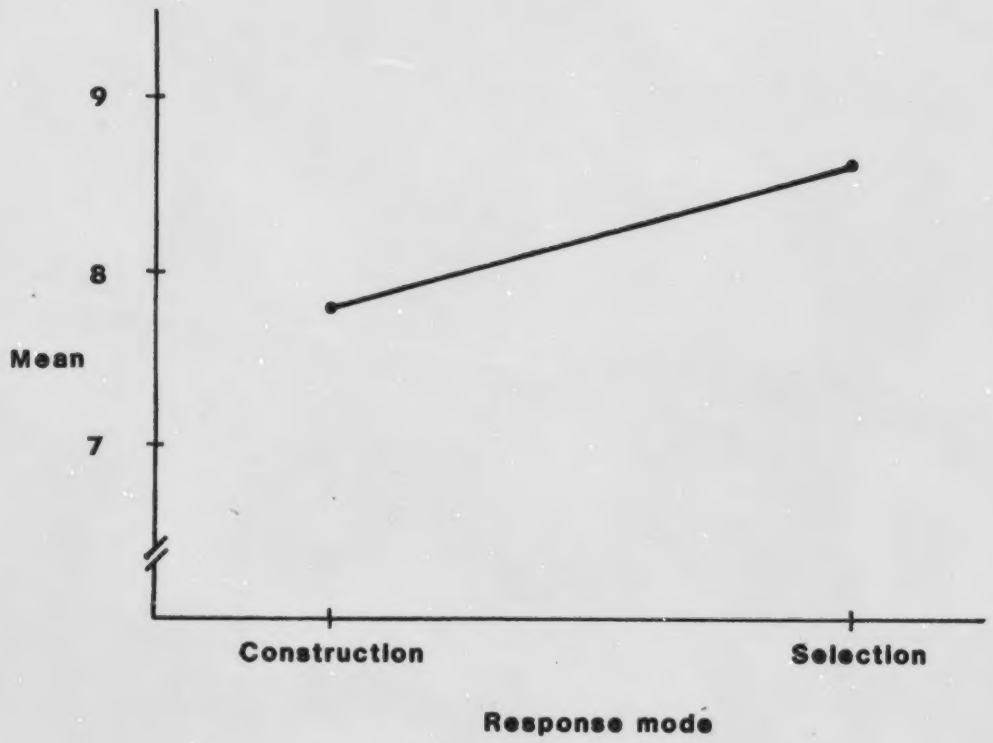


Figure 5: Mean number correct as a function of response mode. Study 3 data

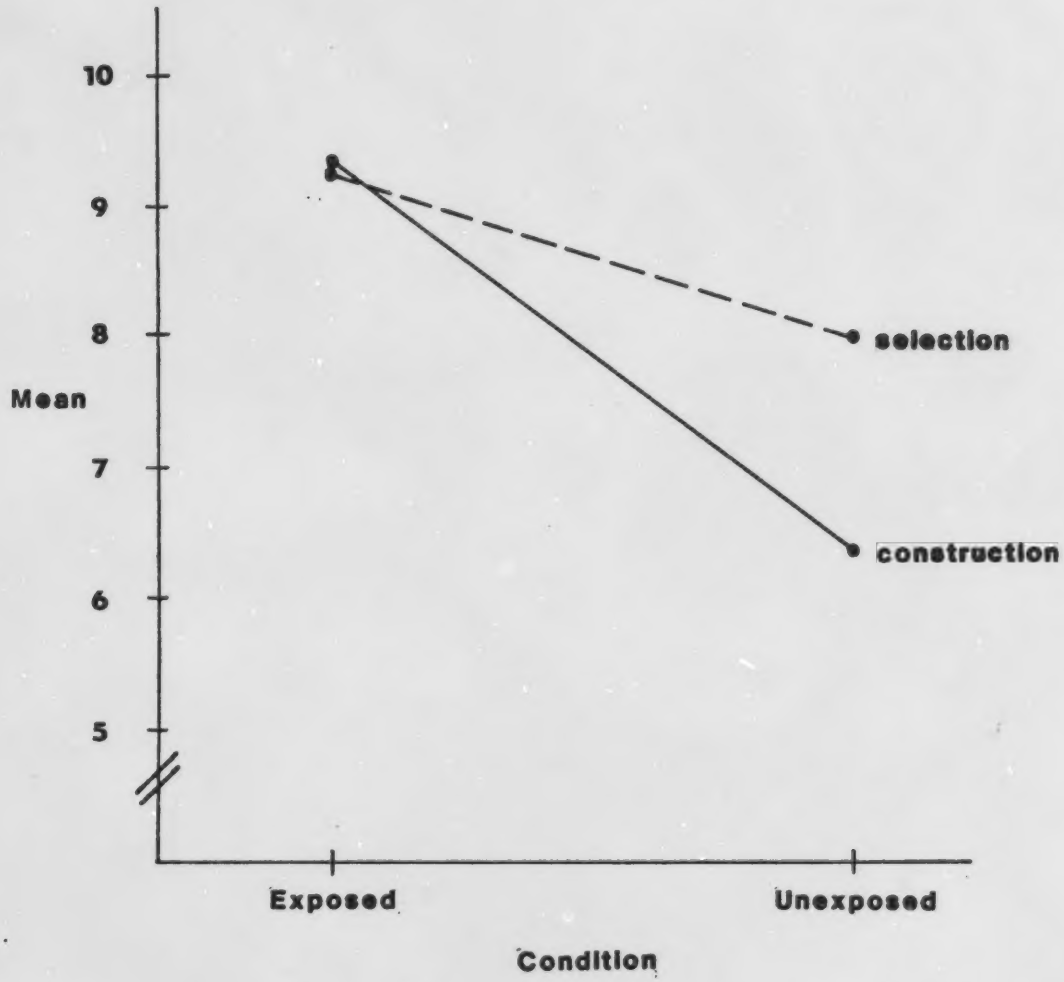


Figure 6: Mean number correct as a function of condition for construction and selection task. Study 3 data

Table 14

Mean Scores by Age, Exposure Condition, and Sex Using a Unified Measure (Response Mode Ignored): Study 3 Data

Age	3		4		5	
Sex	M	F	M	F	M	F
Exposed $\bar{X}$ = 18.54	$\bar{X}$=12.33	$\bar{X}$=14.78	$\bar{X}$=19.56	$\bar{X}$=20.78	$\bar{X}$=21.44	$\bar{X}$=22.33
Unexposed $\bar{X}$ = 14.06	$\bar{X}$=9.22	$\bar{X}$=11.33	$\bar{X}$=15.67	$\bar{X}$=15.00	$\bar{X}$=13.56	$\bar{X}$=14.56
	$\bar{X}$=10.78	$\bar{X}$=13.06	$\bar{X}$=17.61	$\bar{X}$=17.89	$\bar{X}$=20.0	$\bar{X}$=18.44
	$\bar{X}$ = 11.92		$\bar{X}$ = 17.75		$\bar{X}$ = 19.22	

Exp	M	F
	$\bar{X}$=17.78	$\bar{X}$=19.30
Unexp	$\bar{X}$=14.48	$\bar{X}$=13.63

M = 16.30

T shaped display required the following to be scored as correct: (1) a 90° angle had to be preserved between the two constituent bars of the T, (2) the horizontal bar had to be bisected by the vertical bar exactly at its midpoint, (3) the bars had to be in contact with each other, (4) the proper colored bars had to be assigned to the vertical and horizontal positions, and (5) the T had to be properly oriented within the square white reference frame provided. Despite the stringent scoring criteria, three year olds still averaged 6 and 4 items correct out of the 14 for the exposed and unexposed conditions, respectively. This observed competence of the three-year-old subjects replicates the findings of the first two studies.

The higher reliability noted for the construction task was somewhat of a surprise. It was expected that the selection task would prove the most precise of the two measures due to the almost total elimination of the motor component in performance. It appears, however, that the increased probability of guessing a correct response introduces more random error into the measurement process, reducing precision. The selection task was also found to be easier than the construction task, however, its advantage was limited primarily to the unexposed condition. This is probably due to two factors. First, the unexposed condition, due to its increased difficulty, is more likely to generate guessing. The probability of a chance correct response is higher in the selection task. Second, as in many other areas, recognition is probably an easier task than is recall.

E. Study 4

1. Research Objective

The first three studies in this series examining low-level skills revealed (1) the extent to which young children could exhibit comprehension of Euclidean features, (2) whether they could be equivalently successful with both two- and three-dimensional materials, and (3) the extent to which performance differed when functioning at both a percpetual and conceptual level. At the same time the studies had provided the opportunity, (4) to assess two response alternatives, i.e., a construction task and a selection task, (5) to evaluate the unity of items tapping the Euclidean domain, and (6) to experiment with various scoring systems and analyses.

An important question still remained unanswered, however. For example, it was still unknown whether children could cross dimensions when represent-

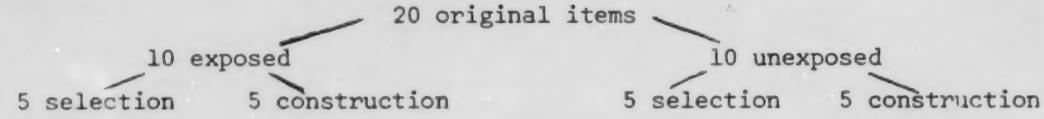
ing Euclidean features. Would subjects be able to translate stimulus material presented in either two or three dimensional displays into its equivalent in the alternative dimension? This then became the additional variable added in the final study.

2. Method

a. Subjects. One hundred and eight children ranging in age from 3 to 6 years served as participants in the study and were again divided into three age groups. The mean age of the youngest group was 3 years 7 months, of the middle group 4 years 5 months, and of the oldest group 5 years 4 months. Subjects were drawn from the same population pool described earlier with equal numbers of boys and girls represented.

b. Materials. The same type of stimulus displays were employed in this final examination. However, since hierarchical analyses had repeatedly indicated the independence of steps 3 and 5 from the remainder of the hierarchical sequence, the eight items representing these steps were eliminated. Therefore, the original 28 item pool was reduced to 20 items representing five hierarchical steps. Four parallel items still remained at each skill level after this paring.

Since we still wished to compare performance on a construction task with that on the selection form, and to assess perceptual and conceptual functioning, this pool of 20 items was further subdivided. Half of the items (n=10) were to serve as items in the exposed condition while the remaining half were to become unexposed items. These sets of ten were further divided on the basis of response mode; five were selection items and five were construction. These divisions are represented diagrammatically as follows:



Within any set of 4 parallel items, the first was a construction-exposed, the second, a construction-unexposed, the third, a selection-exposed, and the fourth, a selection-unexposed. This order of item type was fixed across all five levels and all children received the identical sequence of items.

Two complete sets of materials structured in the described manner were utilized. In one set, all the stimulus displays were constructed of the

two-dimensional paper strips described earlier; the corresponding response boards and construction items were of three-dimensional blocks. The second set was just the reverse.

c. Scoring. Items were scored as either correct or incorrect, since the two types of response procedures were to be directly compared. Each child therefore received 4 scores: (1) construction exposed, (2) construction unexposed, (3) selection exposed, (4) selection unexposed. Each score had a range of 0-5 possible points.

d. Procedure. Children were randomly assigned to two experimental groups. One group received two-dimensional stimuli and three-dimensional response devices; the other received three-dimensional stimuli and two-dimensional response devices.

Each child was assessed in one experimental session rather than the two utilized in earlier studies. Since both unexposed and exposed items were now incorporated into one testing device, the need for two sessions was eliminated. In all other respects procedure paralleled the first three studies.

3. Results

Data were analyzed in a 3(age)x2(sex)x2(direction of transformation 3D to 2D versus 2D to 3D)x2(condition)x2(response mode) analyses of variance. The first three factors served as between factors and the last two were within variables. Significant main effects were detected for age ($F_{1,96} = 49.30$, $p < .001$), and for condition ($F_{1,96} = 134.01$, $p < .001$) but not for either sex or response mode. Significant first-order interactions were revealed for response mode x condition ($F_{1,96} = 38.9$, $p < .001$), and age x condition ($F_{2,06} = 5.66$, $p < .01$). A significant three-way interaction (response x condition x direction) ($F_{1,96} = 4.17$, $p < .05$) was also detected, as was the five-way interaction ($F_{2,56} = 3.42$, $p < .05$). The summary of the analysis of variance is presented in Table 15. Graphs of the main effects and interactions are included in Figures 7 through 11.

Post hoc analyses indicated that five year olds performed significantly better than four year olds ($p < .01$) who in turn surpassed three year olds ($p < .01$). Examination of the condition x response mode interaction effect indicated that in the exposed condition children performed better with a construction response ($p < .01$) whereas in the unexposed condition the

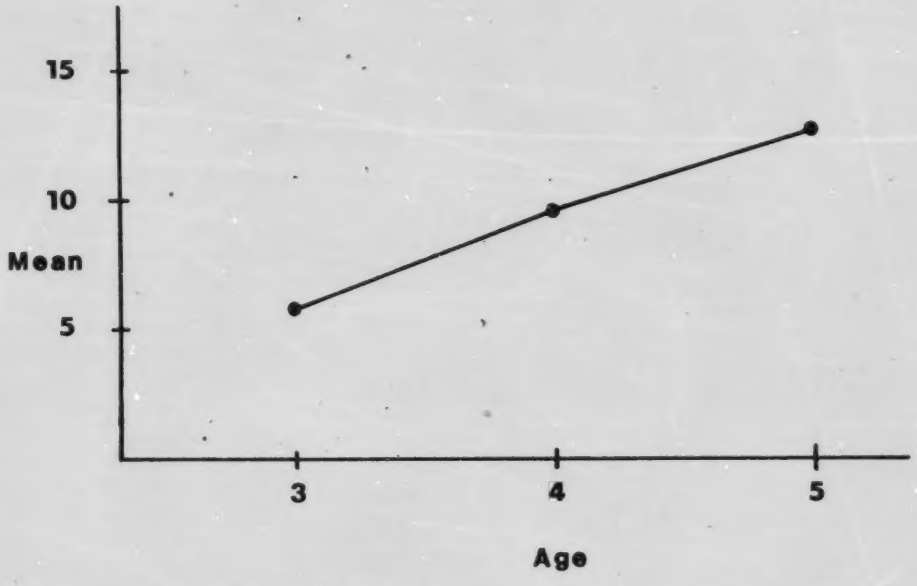


Figure 7: Mean number correct as a function of age. Study 4 data

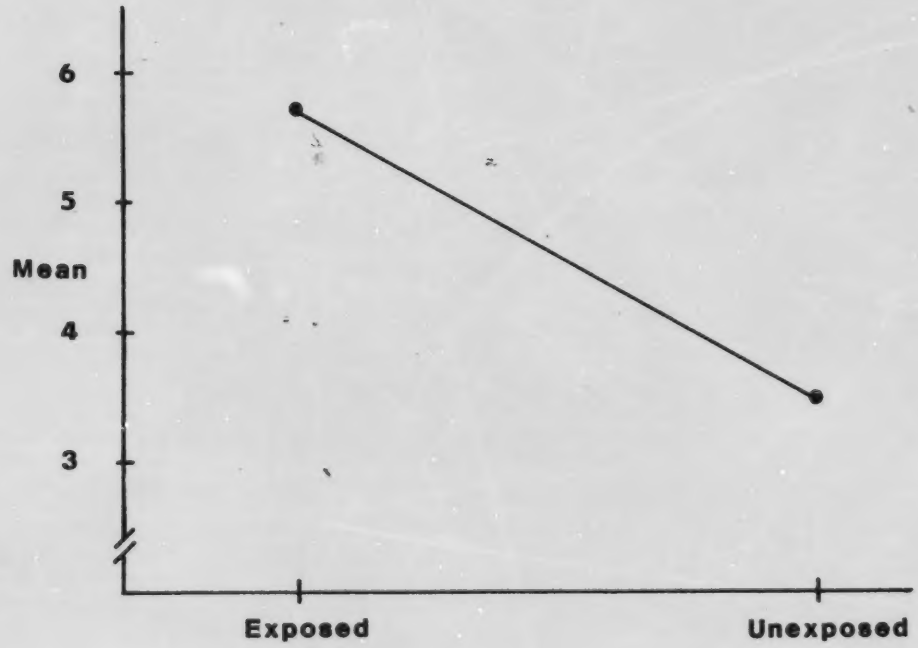


Figure 8: Mean number correct as a function of condition. Study 4 data

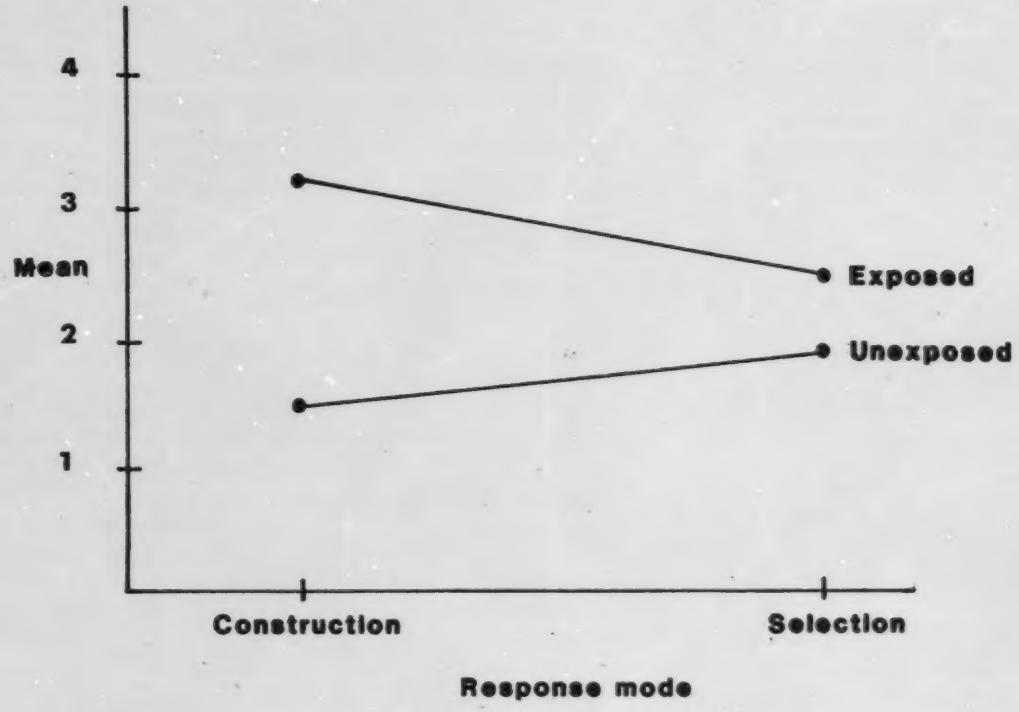


Figure 9: Mean number correct as a function of response mode & condition. Study 4 data

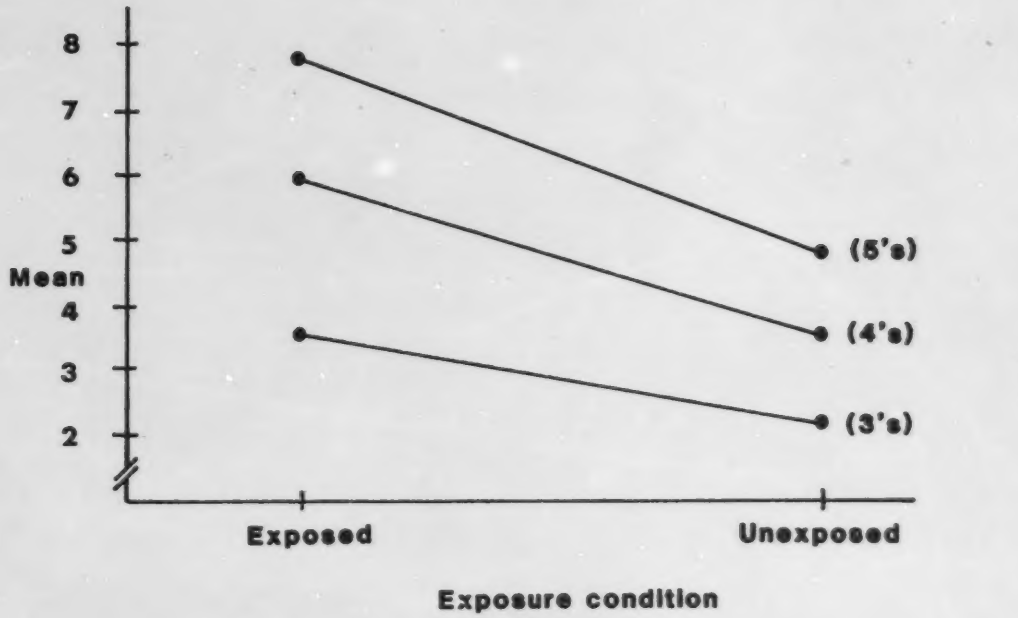
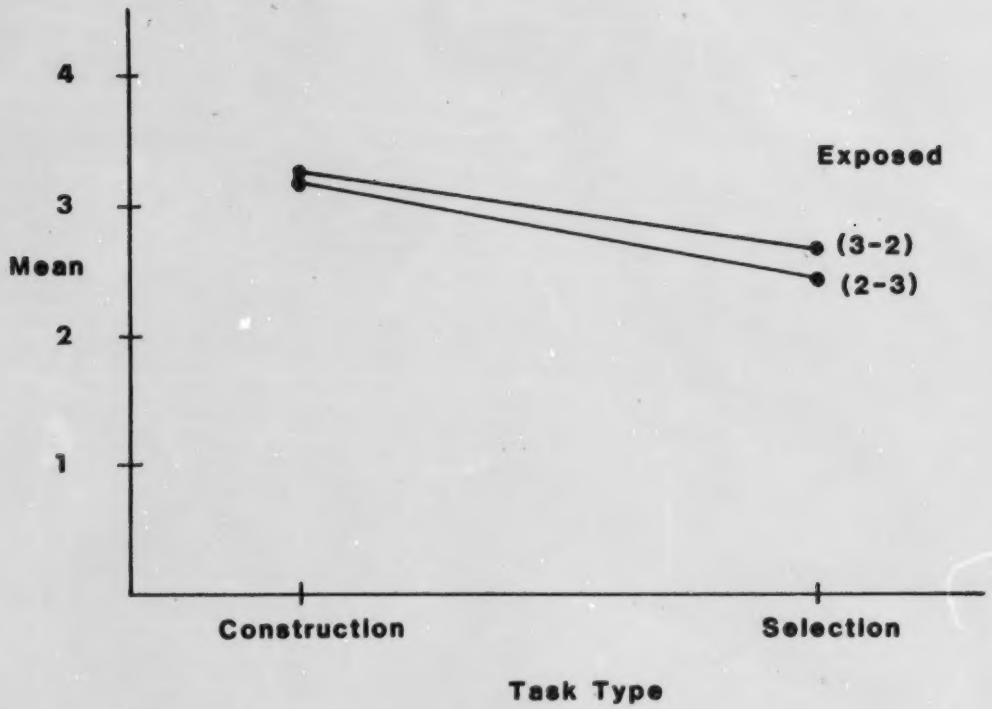


Figure 10: Mean number correct as a function of condition and age. Study 4 data



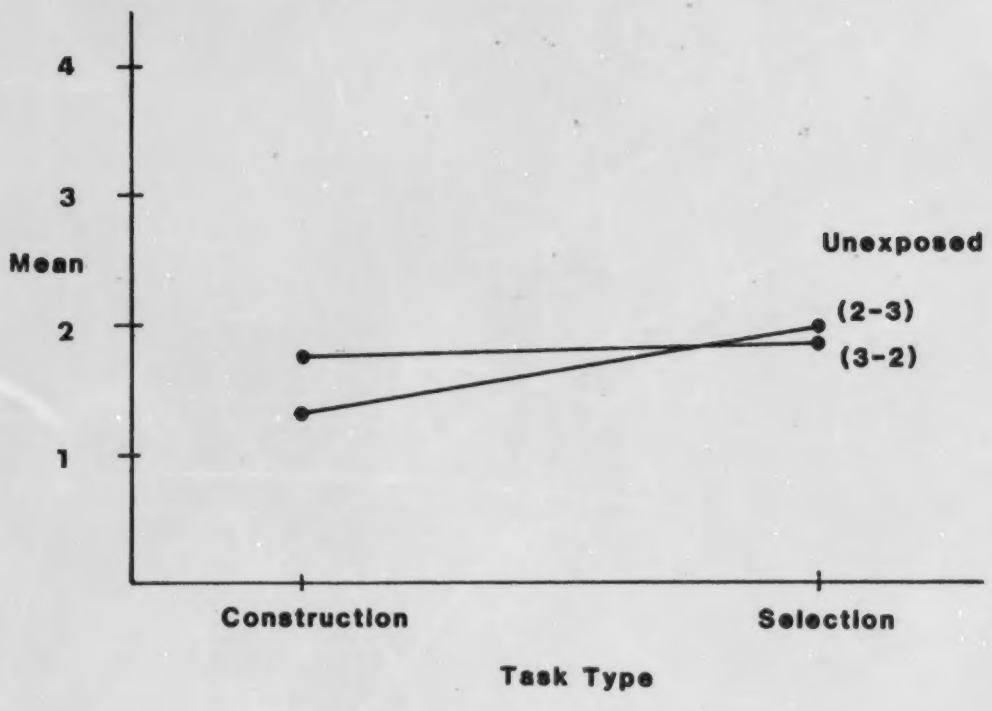


Figure 11: Mean number correct as a function of Condition x Task Type x Direction of transformation. Study 4 data

Table 15

Summary of the Analysis of Variance: Study 4 Data

Source	df	MS	F	p
R (response)	1	2.22	2.22	ns
C (condition)	1	136.69	134.01	< .001
A (age)	2	107.96	49.30	< .001
G (sex)	1	.19	.09	ns
D (direction)	1	2.22	1.01	ns
RxC	1	29.56	38.9	< .001
RxA	2	1.39	1.39	ns
CxA	2	5.77	5.66	< .01
RxG	1	.02	.02	ns
CxG	1	.06	.06	ns
AxG	2	1.89	.86	ns
RxD	1	.67	.67	ns
CxD	1	.02	.01	ns
AxD	2	2.92	1.33	ns
GxD	1	3.50	1.60	ns
RxCxA	2	.06	.08	ns
RxCxG	1	.19	.25	ns
RxAxG	2	.17	.17	ns
CxAxG	2	.09	.09	ns
RxCxD	1	3.17	4.17	< .05
RxAxD	2	.31	.31	ns
CxAxD	2	.13	.13	ns
RxGxD	1	.11	.11	ns
CxGxD	1	.00	.00	ns
AxGxD	2	.51	.23	ns
RxCxAxG	2	.92	1.21	ns
RxCxAxD	2	.17	.22	ns
RxCxGxD	1	.00	.00	ns
RxAxGxD	2	.21	.21	ns
CxAxGxD	2	.24	.24	ns

Table 15--Continued

Source	df	MS	F	p
RxCxAxGxD	2	2.60	3.42	< .05
S(AGD)	96	2.19		
S(AGD)R	96	1.01		
S(AGD)C	96	1.02		
S(AGD)RC	96	.76		

reverse was true ($p < .01$). Performance in the exposed was still superior to unexposed with both response measures ($p < .01$). Further analysis of the condition x age interaction reiterated the main effects; in all cases fives were superior to fours who surpassed threes ($p < .01$), and exposed proved easier than unexposed ($p < .01$). In the three-way interaction, none of the pairwise comparisons across direction of transformation were significant, nor were contrasts across response mode.

F. General Discussion of the First Series

Though four different studies were conducted at this first level of competence, there were a number of parallels both procedurally and in terms of the results. Procedural parallels including (1) similarity of materials, (2) duplication of individual items, (3) matched verbal directions across studies, and (4) repetitions of the same response modes, would seem to allow for direct comparisons across studies and for comprehensive evaluation.

The first of these general conclusions concerns proficiency and acquisition rate. Children performed very well across the individual studies and in general success rates were high. As a means of making comparisons across studies for descriptive purposes, the means from studies 2, 3, and 4 were transformed and equated to serve as an index of proficiency in terms of percent. The first study was not included since a different scoring system was employed with that initial data. These summary statistics were compiled from the mean success rates of each age group for the two-dimensional materials, the three-dimensional ones, and the transformational tasks. The equated proficiency rates are depicted in Figure 12. A summary across all the studies was similarly compiled and is presented in Figure 13. While it would be questionable to analyze this type of compressed data with formal statistical procedures, the descriptive picture is suggestive none the less.

Inspecting Figure 12 suggests three conclusions. First, the overall proficiency rates across the studies are very high approaching 75% for the oldest children assessed with two-dimensional materials. Second, the success patterns across the age groups are strikingly parallel for all three types of materials. And third, success rates are higher when stimuli and responses are in the same dimension than when a transformation across dimensions is required.

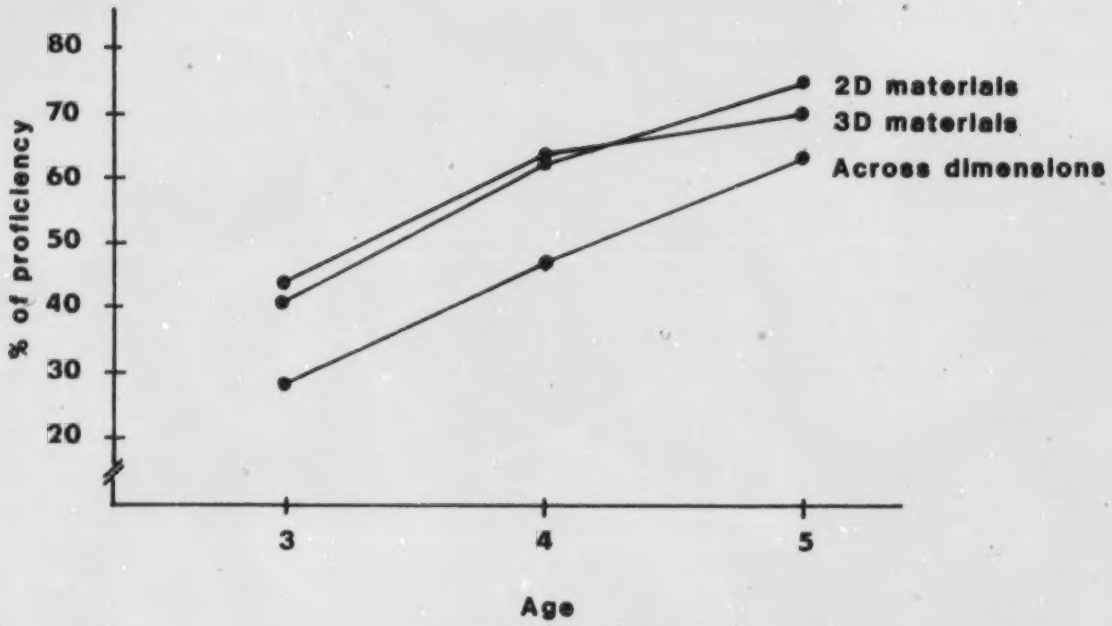


Figure 12: Summary data across studies: General profiency by Material type & age

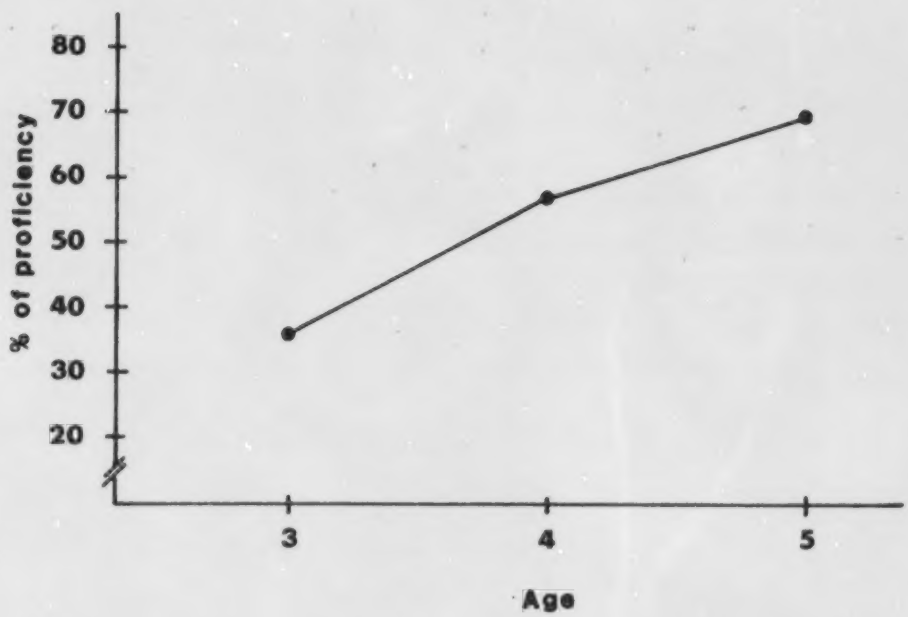


Figure 13: Summary data across studies: General proficiency by age.

If one assumes that the functions depicted in Figures 12 and 13 serve as extrapolated growth curves, the ascending linear nature of those curves is similarly suggestive. Proficiency rates appear to increase quantitatively with age rather than in an all or none fashion or a stepwise progression. This differs, then, from what would be predicted from a cognitive developmental perspective. On the basis of these data, therefore, it appears that both success rates and the nature of increase in those success rates differ from the developmental sequence others would predict for spatial competence (e.g., Piaget & Inhelder, 1956). This point is reiterated in the growth curves based on the first studies results where the data were scored quantitatively.

The same compression technique was employed to compare exposed and unexposed data across studies 2, 3, and 4. These data are depicted in Figures 14 and 15. In all studies, the unexposed condition proved more difficult which reiterates the results of the formal statistical analyses. Again, however, higher proficiency rates are indicated for same-dimensional functioning and lower ones when a cross-dimensional performance is assessed.

Another issue which needs addressing is the one sex difference detected in examining error type in study 1. The results of this errors analysis was not replicated in the subsequent experiments. Interpretation of this finding is ambiguous since no sex differences were revealed across studies on the basis of correct responding. It seems reasonable to assume that the difference occurred simply on the basis of chance and may therefore be meaningless. This conclusion is consistent with the lack of replication of the effect across studies; the other effects were replicated with strikingly parallel results across four studies and 288 subjects.

On the other hand, there were procedural differences between study 1 and the others. Only in the first study was a construction response utilized across the full range of items. The construction response appeared to be both more reliable and more sensitive than the parallel selection device. Perhaps the former measure detected nuances of a child's understanding and errors to which the selection device was insensitive. Although the construction response was employed throughout the series of studies, the number of items assessed in this fashion were fewer. Given

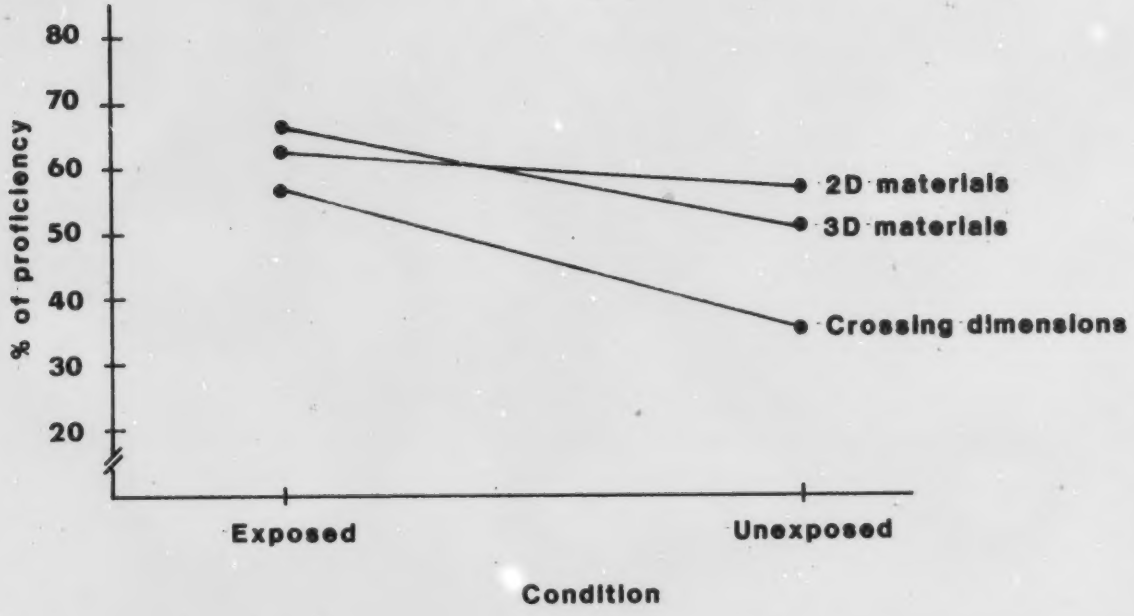


Figure 14: Summary data across studies: General proficiency by material type & condition

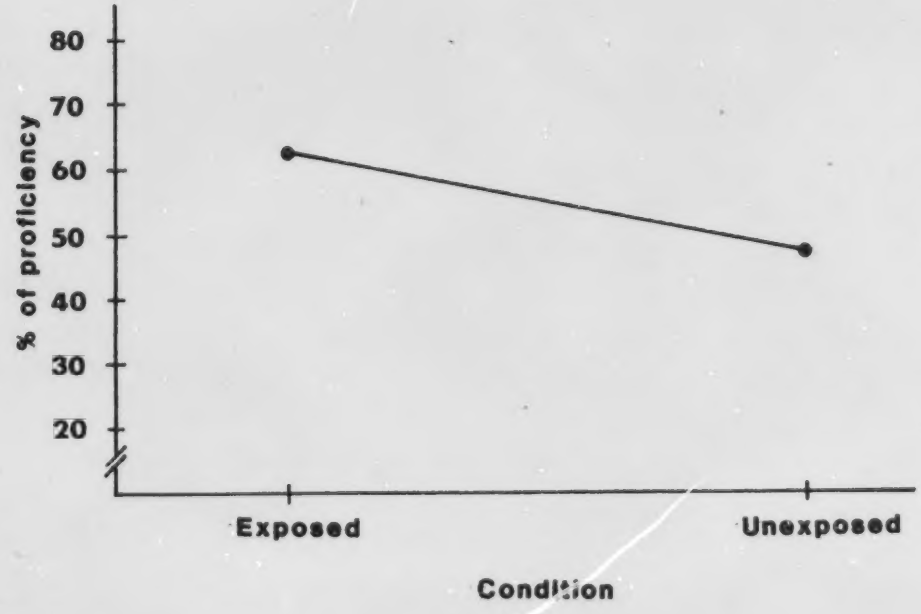


Figure 15: Summary data across studies. General proficiency by condition.

the overall high proficiency rates and the accompanying low error rates, along with the limited items assessed with construction, it could be that insufficient errors were generated thus restricting a sensitive analysis.

If we assume that the sex difference finding was not a chance occurrence, it raises an interesting though unsubstantiated hypothesis. Boys, when erring, frequently did so by reorienting the entire stimulus, disregarding the provided reference frame. Girls, on the other hand, did appear to employ the frame as a cue; they maintained the orientation while disregarding other essential Euclidean features. Perhaps the frame cue was more important for the girls than it was for the boys. It would be interesting to pursue this question when the cueing function of the reference frame is either eliminated or varied.

The final conclusion from this set of data across the four studies is the superiority of the construction response over the equivalent selection one. The sensitivity and reliability of the former has already been noted. Other advantages appear to be a lower frequency of guessing, increased subject involvement with the task, and the greater amount of information revealed. Since the child's actual response is recorded, one achieves a rich source of data for further analysis. Although protocols have yet to be inspected fully, the information exists and can be more extensively reexamined; that is not the case with the selection device.

The quantitative scoring system employed with the construction response also appears to hold promise. First of all, it offers the possibility of describing precisely what features children preserve rather than the rather uninformative correct/incorrect judgment. Second, it can be expanded to include an increasing number of Euclidean characteristics as stimulus displays increase in complexity. Relative difficulty in items can then be reflected quantitatively in the score. Orienting a straight line in a plane is probably a very different operation than is constructing a reference frame and orienting an object in that frame. A simple right-wrong scoring system does not treat the items differentially, however. The quantitative scoring system allows one to effectively weight items on either a logical or empirical basis, again increasing the sensitivity of the dependent measure.

1. Directions for Future Research in Precursor Skills

In this series of studies two crucial Euclidean features were held constant; length of the elements were not allowed to vary and an external reference frame was always provided. These two features are the specific ones assessed with classic Piagetian tasks. This current body of research fills in the knowledge gap between very simple skills such as straight line orientation and the skills assessed by Piaget. However, it should be possible to extend the utilized assessment procedure upward, thereby creating tasks which tap features parallel to those embedded in Piagetian tasks. Scoring for classic Piagetian tasks tends to be an all or none decision separating mature and immature intellectual accomplishments, but degree of immaturity between complete failure and success is not often assessed with these devices. By extending the construction tasks developed from the current research, high Euclidean features could be added while maintaining the quantification of intermediate levels of success.

For example, the ability to conserve distance when creating a Euclidean display would be assessable if subjects were presented with rods varying in length rather than held constant, as was done here. Choosing the correct length rod then becomes an additional, scorable Euclidean feature, for any one item. The score achieved by a subject would reflect all features assessed, including preservation of distance, not simply a dichotomous pass/fail categorization.

A second feature which could be included in an upward extension of these developed tasks is the use of an external reference frame. Provision of the reference frame and its shape is amenable to experimental manipulation. Subjects may perform differently depending on the relevance of the external cueing device. Including this dimension as an experimental variable would serve as one procedure for determining its importance. Quantitative scores across experimenter-provided reference frame conditions could be compared. The present investigators intend to examine these upward extensions in subsequent research, thus expanding the body of knowledge presented here.

IV. Examination of Visual Perspective-Taking Abilities

A. Statement of the General Problem

The next four studies investigate Lohman's (1979a) spatial orientation factor, the ability to anticipate the appearance of a stimulus array from various perspectives. This ability requires coordination of the relationship between the object viewed and a particular point of view. The individual who has constructed a mature, logical spatial reference system understands the nature of perspective relationships, knows that observers in various positions will experience different, unique, and exclusive viewpoints (Salatas & Flavell, 1976), and can even indicate with quantitative precision, the nature of those other views (Fisbein, Lewis, & Keiffer, 1972).

Piaget and Inhelder (1956) examined this kind of visual perspective-taking skill ontogenetically with a miniature landscape scene in which children selected pictorial matches for a succession of perspectives. The obtained results revealed that subjects younger than six were generally unsuccessful, choosing the egocentric alternative consistent with their own perspective. Six-year-old subjects, who did not necessarily respond egocentrically, were still unable to respond accurately, and in fact, correct selections were not common until age nine.

Recent research, however, casts doubt on the apparent visual perspective-taking incompetence of the young child and suggests earlier success. For example, Shantz and Watson (1970) employed a "trick" condition in which perspective changes and positional changes did not correspond. This trick was accomplished by displaying the visual array to the subject, then shielding the array and moving the subject to another position. While the subject changed position, the array was rotated such that the original view was held constant; hence when unshielded, the original perspective was presented again. Results indicated that preschool subjects evidenced a surprise reaction to the trick condition, suggesting possible violation of an expectation for perspective change following locational change. Based on this and a subsequent study (Shantz & Watson, 1970, 1971) the investigators concluded that precursor abilities to the coordination of perspectives may occur at an earlier age than previously predicted.

Liben's (1978b) research with young children supports such a contention by demonstrating that young children can successfully perform perspective-taking tasks if those tasks are not spatially and representationally complex. When the experimenter and subjects were each given glasses with different colored lenses and the subjects were asked to report what color a plain white card appeared both to themselves and to the experimenter, six and seven year olds consistently responded correctly while more than one half of the four and five year olds made no response errors. However, when the perspective-taking task was made more complex by involving spatial representational skills, the four- and five-year-old children exhibited a consistent pattern of errors on even their own viewpoints and it was not until age six that such errors became infrequent. Based on these results, Liben (1978b) discounts the prevailing hypothesis (Modgil & Modgil, 1976) that a child cannot infer another's perspective due to egocentric tendencies and an inability to decenter, and indicates instead the importance of task complexity.

An even more dramatic challenge to an egocentrism explanation of the young child's failure is apparent in Borke's (1975) research with the visual perspective-taking skills of three and four year olds. In this study, children's performance on a display containing three distinct object groups was accurate for all three positions 80 percent of the time. A more complex display with eight object groups and a number of objects in each group yielded correct predictions for all perspectives 79 percent of the time for three year olds and 93 percent of the time for four year olds. On a third display, a duplicate of Piaget and Inhelder's three mountain scene, three year olds gave 42 percent accurate predictions for the three positions, while four year olds were correct on 67 percent of their judgments. Borke concluded that young children's perspective coordination abilities vary as a function of stimulus and task variables rather than of egocentric tendencies, and furthermore that the classic task is too complex a measure for the assessment of early spatial representational skill.

Two specific task variables suggested by Borke (1975) as functionally related to successful perspective-taking performance are level of stimulus complexity and the manner in which children are expected to indicate

correct perspective choices. Although it has been argued that the number of stimulus features presented to the child is not a related variable and should not increase the complexity of the perspective-taking task (Modgil & Modgil, 1976), a careful review of the relevant literature reveals inconsistencies in the results of studies which varied the number of stimulus features. Both Nigl and Fishbein (1974) who compared three, five, and seven object arrays and Liben (1978b) who compared two and three block arrays found that performance did not vary as a function of the number of stimulus features in the array. These results were attributed to the fact that two object arrays provide all the information necessary for establishing front-back and left-right relationships. Additional objects included in the array do not increase the number of spatial relationships because the information which they provide is essentially redundant (Nigl & Fishbein, 1974; Liben, 1978b). If this explanation is accurate, one would expect to find a performance decrement with two object arrays compared with single object arrays, due to an increase in spatial relationships. Such a decrement in performance was found by Fishbein, Lewis, and Keiffer (1972) who reported that preschoolers and kindergartners through third graders success on a visual perspective-taking task decreased when the stimulus array was increased from one toy to three toys. Brodzinsky, Jackson, and Overton (1972), however, did not find a similar difference in the performance of six, eight, and ten year olds between single and multiple object arrays. It may be that older children are not as sensitive to increased task complexity as younger children.

Evidence for the importance of response mode variations is similarly contradictory. A significant performance increment for kindergartners and second graders utilizing a turning response where a duplicate array can be rotated to the appropriate perspective, as opposed to the more frequently employed photograph selection response, was observed by Fishbein, Lewis, and Keiffer (1972). However, this facilitative effect was not found at all levels of age and sex. Borke (1975) did find a positive effect using a rotation response but failed to make a comparison between rotation and other modes of responding. Horan and Rosser (1979) directly compared the effects of a rotation response over photograph selection upon visual

perspective taking with a single stimulus. Three-year-old subjects were able to successfully perform the perspective-taking task given a rotation response while four year olds demonstrated visual perspective-taking abilities given either the photograph selection or rotation response.

A number of questions remain unanswered by the body of research where response mode was varied. Perhaps the rotation response is more facilitative of performance because both the stimulus and response arrays are in the same dimension while the photograph selection response involves a three-dimensional stimulus array and two-dimensional response options. Or, the rotation task could simply be less complex than a photograph selection task, independent of any variations in dimensionality. However, Lohman (1979a) has pointed out that the spatial orientation is the least robust of the identified spatial factors perhaps because subjects' utilize a variety of strategies for problem solving. It could be that alteration of task variables predisposed subjects toward a particular solution approach. Differences in outcomes across studies differing in experimental demands may in fact represent differences in the processing facilitated by those demands. Comparisons across studies which differ in this way may therefore be unwise; they may not be tapping identical spatial abilities. In the next set of studies, we choose to compare responses within a single study holding other experimental demands constant. Perhaps in that way, more information would be revealed as to the relevance of response demands to internal processes.

B. Study 5

1. Research Objective

The first study in this series examined the performance of children on perspective-taking tasks when both the stimulus presentation and the response device were in the same two-dimensional mode. Holding dimension constant, the aim was to directly compare task variables of item type and response type for relative ease of accomplishment. Therefore, three response devices were designed: (a) a rotational task, (2) a constructive task, and (3) a selection task. In addition two stimulus item types were created: (1) those that depicted relations between objects and (b) those that depicted formal Euclidean features.

2. Method

a. Subjects. One hundred and twenty subjects four to eight years of age participated in this study. These subjects formed three age groups containing equal numbers of males and females: Four year olds, $\overline{X}_{age}$ = 4 years 6 months, six year olds, $\overline{X}_{age}$ = 6 years 5 months, and eight year olds $\overline{X}_{age}$ = 8 years 5 months. Subjects were drawn from seven preschools and grammar schools, both public and private, in Tucson, Arizona. These children were from predominantly middle to upper middle class socioeconomic backgrounds.

b. Materials. All stimulus items were depicted in a two-dimensional format. In the familiarization and single object rotation segments, black and white sketches rendered on 8" square cards were employed. Euclidean items were constructed using the strips of colored cardboard paper and white cardboard squares described in Study 1. Response items for the rotation task with both single objects and Euclidean displays consisted of duplicates of the stimulus cards which could be turned. Blank white cards and the appropriate kind and number of cardboard cut-outs were provided for the construction response task. For the selection task, two options, one correct and one incorrect, were represented on large pieces of cardboard (16 and 3/4 x 8" and 12 and 3/4" x 12") divided on the vertical mid-line by 3/4" black masking tape. Cardboard strips were once again used to form the two response choices.

c. Scoring. A coder assisted the experimenter in this task and recorded subjects' responses. Sketches of the single objects rotated to all four positions were displayed on the coding sheet for the single object rotation task. Coders simply marked an X through the perspective indicated by the child. Euclidean rotation and construction responses were drawn by the coder. For the selection response, the coder checked which option the subject had selected on the coding sheet's depiction of the stimulus response card on the basis of placement. Construction and rotation items were scored either correct or incorrect with each correct response earning 1 point. For each of the three response modes on the Euclidean items, there was a total possible score of 9 points for perspective taking, with an additional 9 points for the rotation of single-object items.

d. Procedure. Subjects were divided into two experimental conditions such that each group contained an equal number of subjects of each age and sex. In the exposed condition, the experimenter (E) turned each perspective-taking stimulus array 360°, allowing the subject (S) to view the array from all four sides. E stopped the rotation briefly at each position and then continued turning in a clockwise fashion. Subjects in the unexposed condition did not benefit from actually viewing all sides and only directly perceived arrays from the 0° rotation position.

Prior to formal assessment, a familiarization task, to acquaint subjects with the concepts of viewing different perspectives, was conducted. The first part of this task emphasized that E and S saw different objects depending upon their viewpoint. E brought out a card with a line drawning of a tree depicted on one side and a rabbit on the opposite side. S was shown both sides and then the card was placed flat on the table with the tree side exposed. S was asked what S saw and what E saw. S was asked what S saw and what E saw. S was also asked if S or E saw the rabbit. Then E placed the card in an upright vertical position with the rabbit facing S. S was asked what S saw and what E saw and if S saw the tree and if E saw the rabbit. E emphasized that "We don't always see the same things."

The next segment focused on seeing different perspectives of the same picture. A drawing of a turtle was placed flat on the table and E pointed out various parts of the turtle. Then a divided card was placed along the turle's horizontal midline such that the picture was cut in half; S and E could therefore view only half of the turtle stimulus. S was asked what part of the turtle was seen by S and what part was seen by E. The divider was then removed and the turtle card was rotated 180°. S was then asked what part of the turtle was closest to S and what part was closest to E.

A line drawing of a dog was presented and E again pointed out its structural features: Feet, back, tail, and head. S was asked if E and S saw the same thing and what parts of the dog were closest to S and to E. Finally, E moved sequentially through the 270°, 180°, and 90° rotation position. At each position E turned a duplicate of the stimulus card to demonstrate to S what perspective E was seeing at each position.

Following the familiarization task, line drawings of a horse, car, and house were employed to measure S's comprehension of the array's appearance from the 90°, 180°, and 270° rotation positions. E changed positions in a fixed, but randomly derived, order for all remaining trials. Furthermore, with each position change, regardless of item type or response mode, E first asked S "Do you and I see the picture the same way?" and was asked to show the coder, seated directly to S's right, what E was seeing. Object drawings were responded to only in the rotation response mode while parallel Euclidean arrays were sequentially responded to by rotation, construction, and selection response modes.

The 90°, 180°, and 270° rotation positions were assessed in a random order for each stimulus array before a new stimulus was presented. Thus, there were nine perspective-taking responses for Euclidean items in each response mode, while there were nine perspective rotation responses for the single-object items.

Rotation of the Euclidean arrays immediately followed rotation of the single-object items. However, both the construction and selection response modes were modeled with Euclidean arrays not included in assessment. E moved to the 180° position for modeling. During construction modeling, E employed cardboard cut-outs and constructed on the blank white card placed in front of S, the perspective of the array that E was viewing. Assessment through the construction response mode began with E presenting the first stimulus card to S. S was told by E to "Make a picture to show the coder what I am seeing from here." At the end of each construction response, E removed S's materials before moving to a new position or presenting the next stimulus card.

Following the construction response mode, E modeled the selection response. A modeling stimulus item was presented followed by a card depicting two possible options. E pointed to the card which showed how E was "seeing the picture" from the 180° position. Following subject assessment in this selection response mode, the experimental session was terminated.

3. Results

The correct response data from the Euclidean items were examined through a 3(age)x2(sex)x2(exposure condition)x3(position)x3(response mode) analysis of variance with repeated measures on the last two factors.

A summary of this analysis is displayed in Table 16. Significant main effects were found for age ($F_{2,108} = 12.69$, $p < .001$), position ($F_{2,216} = 16.39$, $p < .001$), and response mode ($F_{2,216} = 160.33$, $p < .001$). Newman-Keuls post hoc analyses of these effects indicated that eight year olds performed significantly better than six year olds ($p < .01$) who performed better than four year olds ($p < .05$). Perspective-taking performances from the 90° and 270° positions proved to be more difficult than those from position 180° ($p < .01$). The selection response mode facilitated correct perspective taking the most, followed by the rotation response mode ($p < .01$). Construction responses were the least accurate ($p < .01$).

Significant first-order interactions were found for position by response mode ($F_{4,432} = 4.68$, $p < .005$), position by age ($F_{4,108} = 2.98$, $p < .05$) and response mode by age ($F_{4,108} = 2.98$, $p < .05$). Analyses of the position by response mode interaction depicted in Figure 16 revealed that the 180° position was significantly easier than the 90° and 270° positions for the rotation and selection response modes. While the frequency of correct responses to the 90° and 270° positions did not differ in the rotation response mode, more correct responses were made on assessments of the 270° position in the selection task. Furthermore, in the construction response mode, no differences were found between any pairwise comparison of the three positions.

Figure 17 displays the position by age interaction effect. The performances of six- and eight-year-old subjects on the 90° and 270° positions do not differ and are less accurate ($p < .001$) than these subjects' 180° position performance. However, four year olds do not differ in their correct responses to the 180° and 270° which prove easier than their performance to the 90° position ($p < .05$).

The interaction effect of response mode by age is represented in Figure 18. The results reveal that the rotation task is the best discriminator between the three age groups with eight year olds outperforming six year olds ($p < .01$) who, in turn, do better than four year olds ($p < .05$). While eight and six year olds also differ in their performances by the construction response mode ($p < .01$) and the selection response mode ($p < .05$), six year olds and four year olds do not differ when their perspective-taking abilities were assessed by means of these two response

Table 16

Summary of First Analysis of Variance:

Study 5 Data

Source	df	MS	F	p
P (position)	2	11.21	16.39	< .01
M (response mode)	2	153.92	160.33	< .001
G (sex)	1	6.69	2.72	n.s.
C (exposure condition)	1	.33	.13	n.s.
A (age)	2	31.21	12.69	< .001
PM	4	2.37	4.68	< .005
PG	2	.24	.35	n.s.
MG	2	.73	.76	n.s.
PC	2	.20	.29	n.s.
MC	2	.18	.26	n.s.
GC	1	.58	.24	n.s.
PA	4	2.03	2.98	< .05
MA	4	2.86	2.98	< .05
GA	2	3.91	1.58	n.s.
CA	2	.07	.03	n.s.
PMG	4	.45	.88	n.s.
PMC	4	.62	1.23	n.s.
PGC	2	.14	.21	n.s.
MGC	2	.32	.33	n.s.
PMA	8	.90	1.80	n.s.
PGA	4	1.37	2.01	n.s.
MGA	4	.97	1.01	n.s.
PCA	4	.92	1.35	n.s.
MCA	4	.54	.56	n.s.
GCA	2	.58	.24	n.s.
PMGC	4	.36	.71	n.s.
PMGA	8	.77	1.51	n.s.
PMCA	8	.44	.86	n.s.
PGCA	4	.41	.60	n.s.

Table 16--Continued

Source	df	MS	F	p
MGCA	4	1.59	1.66	n.s.
PMGCA	8	1.09	2.14	< .05
S(AGC)	108	2.47		
S(AGC)p	216	.68		
S(AGC)M	216	.96		
S(AGC)PM	432	.51		
Total	1079			

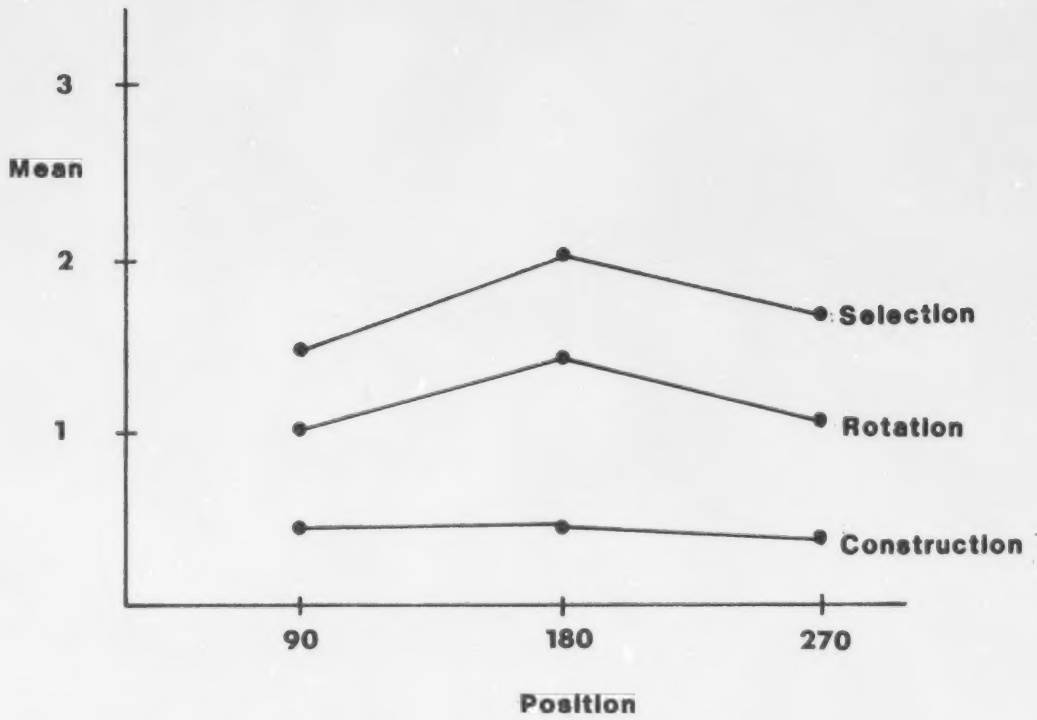


Figure 16: Mean number correct as a function of position x response mode - Study 5 data

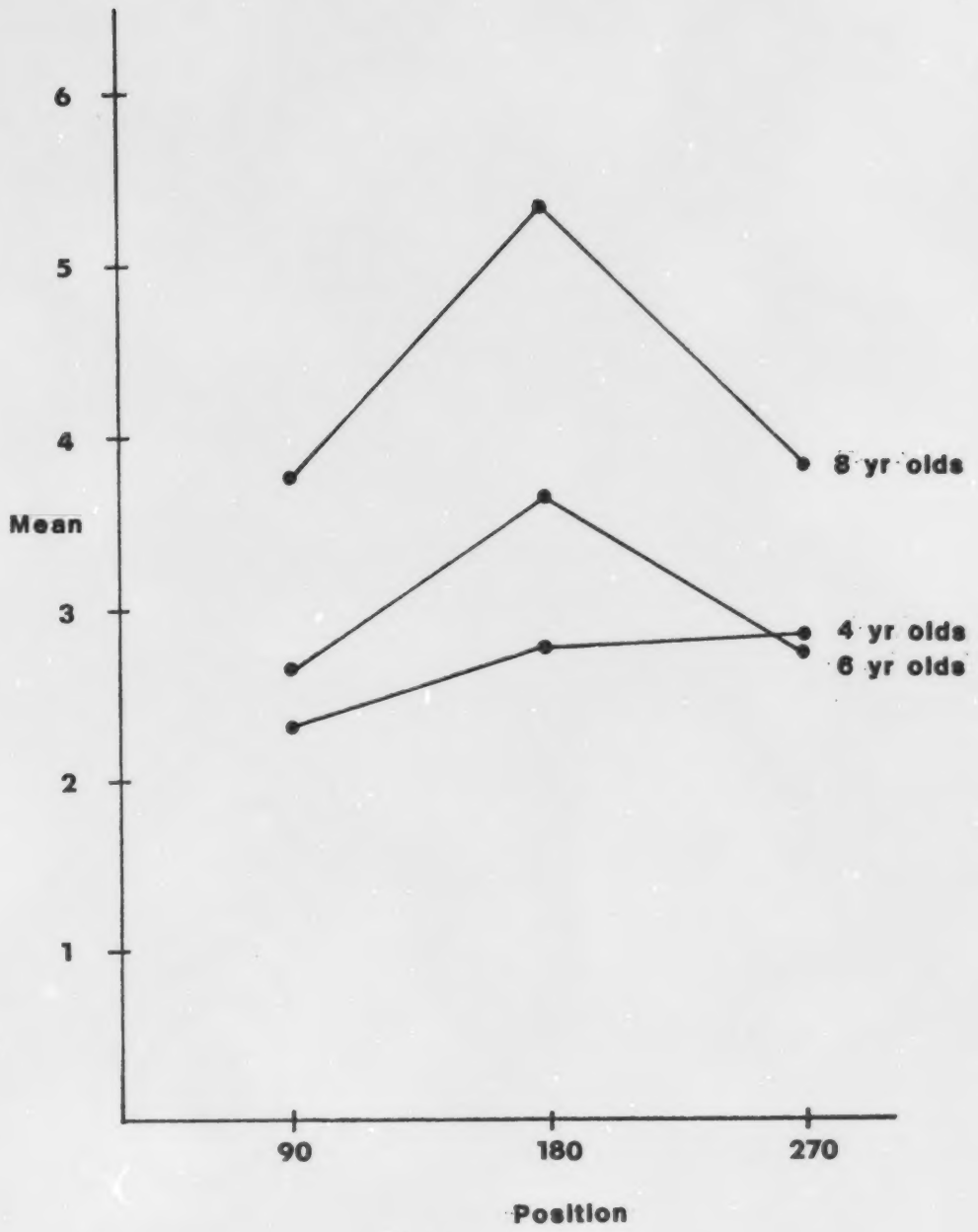


Figure 17: Mean number correct as a function of position x age. Study 5 data

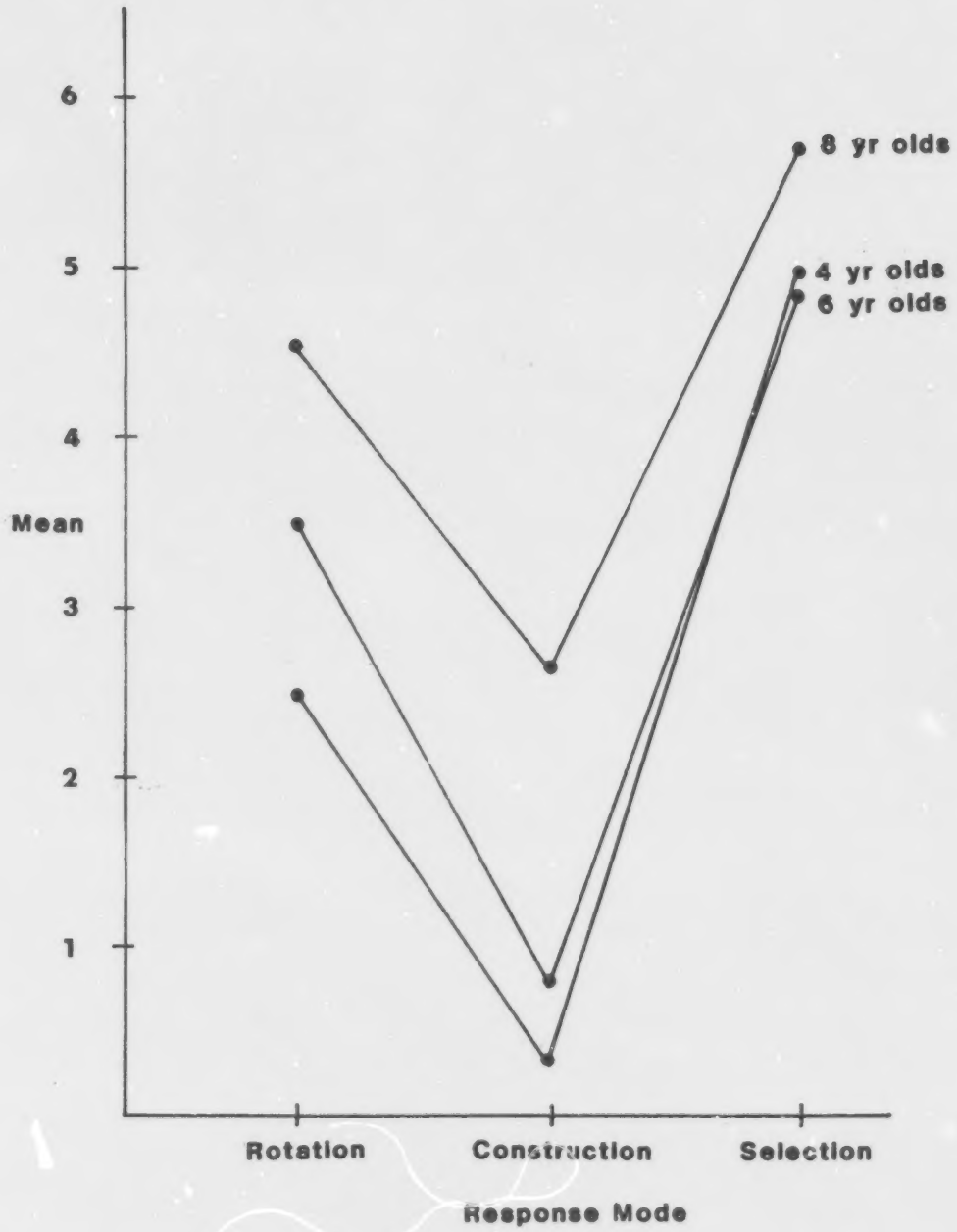


Figure 18: Mean number correct as a function of response mode x age. Study 5 data

modes. Eight year olds, however, did differ from four year olds with $p < .01$ and $p < .05$, respectively.

A second correct response analysis was conducted to compare the Euclidean rotation responses of subjects across positions to the nine single object rotation responses at the three positions. A 3(age)x3(position) x2(item type) analysis of variance, summarized in Table 17, was performed with repeated measurement of the last two factors. A main effect was found for age (F 2,117, 8.49, $p < .001$) which revealed, when further analyzed, that six and eight year olds performed significantly better than four year olds ($p < .01$) but did not differ significantly from each other. Non-Euclidean items proved to be easier than Euclidean ones (F 1,117, 48.49, $p < .001$). There was also a significant main effect for position (F 2,234, 32.94, $p < .001$) which showed that the 90° and 270° positions, which did not differ from one another, were more difficult than the 180° ($p < .01$).

The item type by position interaction (F 2,234, 25.73, $p < .001$) depicted in Figure 19 shows that the 180° position was significantly easier than the 90° and 270° position for the non-Euclidean items only ($p < .01$). There was no differential performance among positions with the Euclidean items. This first-order interaction further reveals that non-Euclidean items are easier than Euclidean items only at the 180° position. However, a second-order interaction of age by item type by position (F 4,234, 2.79, $p < .05$), represented in Figure 20, discloses that eight year olds are restricted to the 180° position in their superior performance with non-Euclidean items; four- and six-year-old subjects perform better with non-Euclidean items at both the 180° position ($p < .01$) and 90° position ($p < .05$).

4. Discussion

Despite the complexity of the design and the statistical significance of a number of interactions, a number of unambiguous conclusions are suggested by these data. First, the age effect is clear with an obvious superiority for the eight year olds, intermediate performance for the six year olds, and the least success associated with four year olds. While this age differential is not particularly surprising, its linear nature suggesting a quantitative increase in proficiency with age is noteworthy. Second, the response modes differ significantly with the selection mode

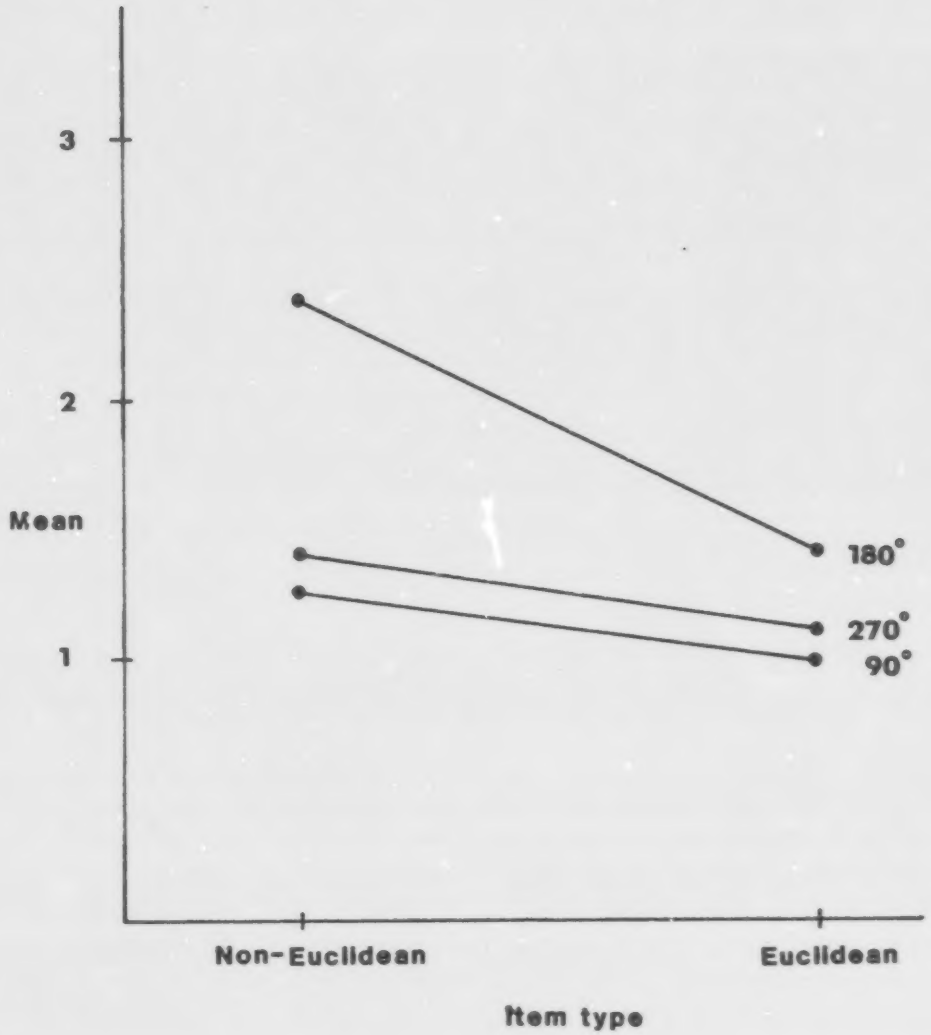


Figure 19 Mean number correct as a function of position x item type. Study 5 data

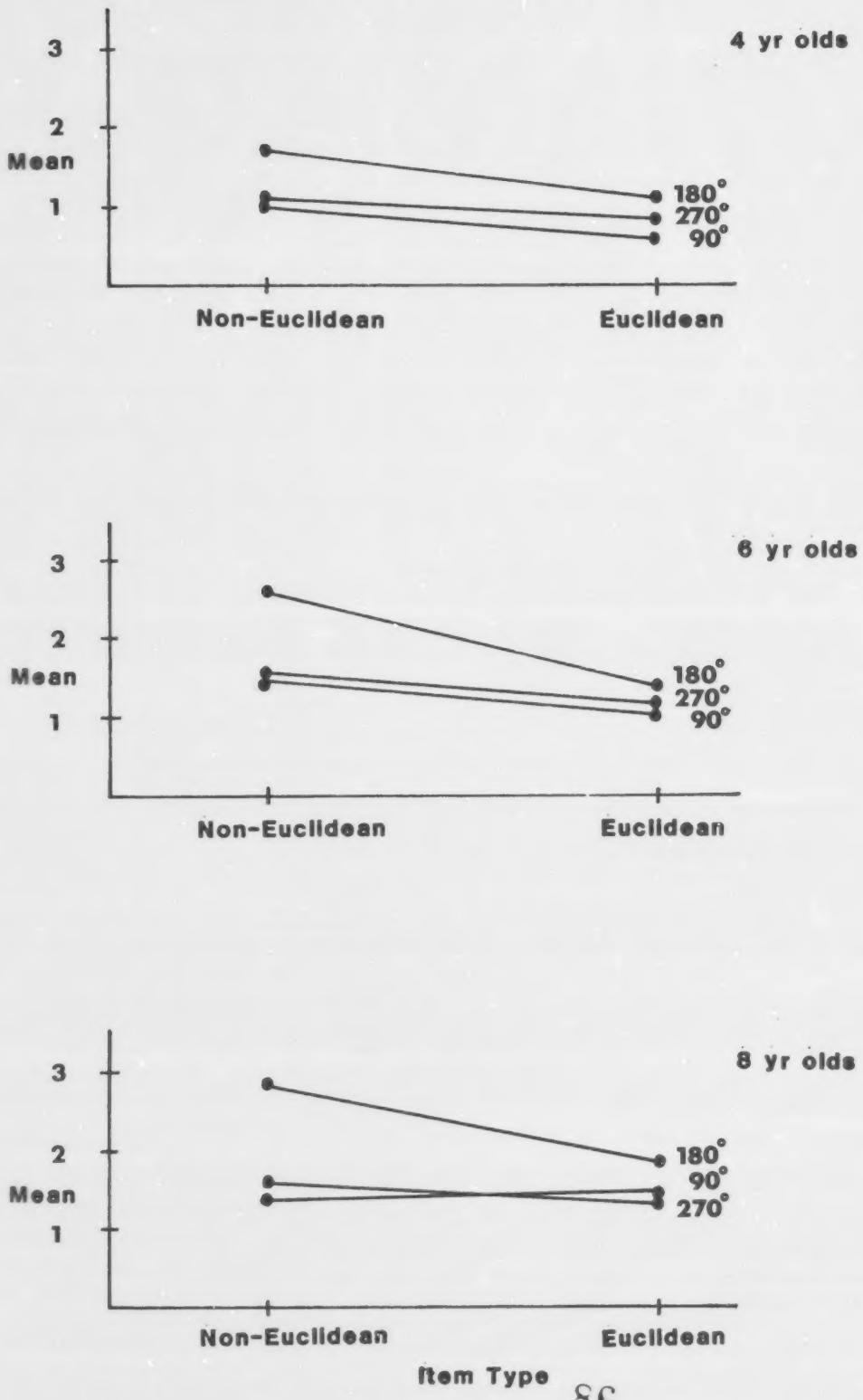


Figure 20: Mean number correct as a function of age x item type x position. Study 5 data

Table 17

Summary of the Second Analyses of Variance: Study 5

Source	df	MS	F	p
A (age)	2	28.18	8.49	< .001
S(A)	117	3.32		
B (item type)	1	44.13	48.49	< .001
AB	2	1.47	1.62	ns
S(A)B	117	.91		
C (position)	2	41.45	32.94	< .001
AC	4	1.26	1	ns
S(A)C	234	1.26		
BC (item type x position)	2	8.85	25.73	< .001
ABC	4	.96	2.79	< .05
S(A)BC	234	.344		
Total	719			

proving easier than rotation which in turn is more facilitative than construction. And third, it seems the 180° position is easier than the two side positions which are of equivalent difficulty.

The ease of the selection mode is not surprising since, with only two response options, correct responding simply on the basis of chance can be expected. In fact the overall mean for the selection mode ($\overline{X}$ = 5.16) is not that much higher than predicted on the basis of chance alone. The apparent relative ease of the rotation response, however, is not so easily explained on the basis of chance alone. The mean ($\overline{X}$ = 3.51) is higher than predicted on the basis of chance even if one assumes a 25% probability of correct response with guessing alone. That assumption presupposes that only four response options are available corresponding to the four sides of the display. In actuality this figure is probably an overestimation since the rotation device could be turned to any angular position by the subject but the scoring criteria was in fact quite rigid. It would seem, therefore, that chance differences alone are an insufficient explanation for the facilitative effect noted for the rotation mode.

Flavell (1976) has proposed two levels of perspective taking. Level 2 is that associated with mature understanding when the individual knows perspectives differ and can determine the nature of object relationships from a variety of viewpoints. At Level 1, however, the subject may in fact know that perspectives do in fact differ but deciphering the precise nature of the spatial relationships is not yet manageable. Level 1 is a developmental precursor to Level 2, and one could conjecture that tasks requiring only Level 1 abilities would prove easier than those necessitating Level 2 comprehension.

To succeed on the construction task, a child would have to possess Level 2 abilities. The object relationships must be noted, preserved, and appropriately recreated by the child. The rotation task does not necessarily require an equivalent level of sophistication. In the latter task, despite the complexity of the display, the subject need only focus on <u>one</u> object in the set and position the response device on the basis of that subject alone; coordination of the relationship <u>between</u> objects need not occur. Borke (1975) contends that the rotation task is more appropriate for young children than the classic one and permits them to demonstrate

their ability to decenter. It appears to the present authors, however, that the rotation task instead taps a lower level skill which may not even involve the coordination of spatial relationships. Direct comparisons of studies which employ a rotation response with examinations using other procedures would, therefore, seem unwise.

One can draw support for this condition from the significant position x mode interaction. Regardless of the position of the view requested, a construction response necessitates a preservation of at least one spatial relationship between two objects, e.g., (A)—(B). It seems logical that irrespective of the view which must be constructed, the task would be equally different; no difference was detected between 90°, 180°, and 270°, perspective constructions. With both selection and rotation tasks, however, subjects need only focus on (A) and ignore (B). Then in the 180° position perspective, subjects must either point to or turn to the option that presents A backwards from the egocentric position. The 180° task, therefore, eliminates the need for dealing with only left-right relationship. Others (Nigl & Fishbein, 1974) have suggested that right-left relationships can be problematic for young children. If the 180° request avoids the problem of left-right coordination, then it could prove easier than either a 90° or 270° request at least for responses focusing on one object. In this study, the 180° position was significantly easier with both the rotation and the selection tasks.

In conclusion, it seems that young children can succeed on perspective-taking tasks, but the conclusion must be qualified: (1) the performance of youngsters in this study does not support the notion that young children display pervasive egocentrism and therefore are consistent with Borke's (1975) conclusions, (2) young children succeed better on tasks which allow them to focus on one aspect or object and a spatial arrangement rather than to coordinate spatial relationships, (3) the rotation mode approach does not necessarily tap a child's ability to coordinate relationships and therefore may be a poor choice when that level of sophistication is of interest.

C. Study 6

1. Research Objective

The next study examined many of the same effects as the last. However, whereas study 5 investigated perspective-taking performances when the

stimulus and response materials were two-dimensional, study 6 focused on these effects when three-dimensional stimulus and response items were employed.

2. Method

a. Subjects. Half of the children in each age and experimental group who participated in study 5 were included in study 6 (N=60). The subjects were once again divided into three age groups of four year olds ($\bar{X}_{age}$ = 4 years 7 months), six year olds ($\bar{X}_{age}$ = 6 years 5 months) and eight year olds ($\bar{X}_{age}$ = 8 years 5 months). There was an equal number of males and females in each age group.

b. Materials. For the non-Euclidean object trials, four toy vehicles (i.e., a blue truck, red van, white sports car, and green Volkswagen "bug") and four toy animales (i.e., an orange lion, yellow hippopotamus, green donkey, and blue elephant) were employed. The objects could be arranged in a variety of complexities depending on the number of them included; four items were designed with each set of toys. Euclidean materials were the same as those described in study 3, again with four levels of stimulus complexity.

A section of green turf-like carpet was the stage upon which the non-Euclidean arrays were displayed. Subjects were given an identical piece of carpet for construction responses and the duplicate arrays employed in rotation were also placed on this piece. White cardboard bases were used for the Euclidean items; with the rotation tasks, a duplicate array was available while the construction task parallels those described earlier. A lazy Susan device was placed beneath the stages in the rotation responses.

c. Scoring. Coders again assisted the experimenter in this study. The coder determined if the child's response precisely matched the correct response depicted on the coding sheet. If there were no errors, one point was assigned. If all of the items were arranged relative to each other, as the items in the display, but with one item facing in the wrong direction, the child's response was considered incorrect. Thus, a high performance criterion was established. Performance scores were computed for both the object items and Euclidean trials which were further subdivided

into rotation and construction responses for each item type.

d. Procedure. To insure counterbalancing subjects received precisely the same treatment with the exception that half of them received vehicles before animals during the object trials while the other half were presented with animals followed by vehicles. Since the exposed condition preceded the unexposed in both the non-Euclidean and Euclidean trials, this meant that some subjects were exposed to all four sides of vehicle arrays but not of animal arrays while the reverse was true for the subjects first presented with animals. As in study 5, exposure consisted of the experimenter (E) turning the stimulus array 360° clockwise and pausing at the 90°, 180°, and 270° positions. In this study, however, exposure condition was treated as a within-subject control procedure rather than as a between-subject experimental factor.

All children proceeded through the experiment in the same sequence. First, four object arrays (either vehicles or animals) were presented in the exposed condition. Second, four object arrays of the other type were presented in the unexposed condition. A construction response was first requested and next, a rotation response followed by another construction and then rotation response. Subsequent responses followed this repetitive order throughout the object and Euclidean tasks, both exposed and unexposed.

3. Results

The data were examined through a 3(age)x2(sex)x2(response mode) x3(position)x2(item type) analysis of variance with repeated measurement of the last three factors. Exposure condition was not included as a factor since it had not had a significant effect in study 5. A summary of this analysis appears in Table 18. Significant main effects were found for age ($F_{2,54} = 10.46$, $p < .001$), response mode ($F_{1,54} = 158.92$, $p < .001$), and item type ($F_{1,54} = 12.65$, $p < .001$). As might be expected, Newman-Keuls post hoc analyses revealed that eight year olds performed better than six year olds who outperformed four year olds. The rotation response mode was more facilitative of correct responding than the construction response mode, while non-Euclidean items proved more difficult than Euclidean items.

A significant interaction effect for response mode by position depicted in Figure 21 revealed that positions 90° and 270° did not differ from one another in either the rotation or construction response mode. However,

Table 18

Summary of Analysis of Variance: Study 6 Data

Source	df	MS	F	p
P (position)	2	.62	.72	n.s
M (response mode)	1	352.80	158.92	< .001
T (item type)	1	12.27	12.65	< .001
A (age)	2	105.54	10.46	< .001
G (sex)	1	3.76	.37	n.s.
PM	2	9.90	16.78	< .001
PT	2	.48	1.02	n.s.
MT	1	22.05	25.34	< .001
PA	4	.79	.92	n.s.
MA	2	3.90	1.76	n.s.
TA	2	4.68	4.78	< .05
PG	2	.47	.55	n.s.
MG	1	1.42	.64	n.s.
TG	1	1.25	1.29	n.s.
AG	2	25.15	2.49	n.s.
PMT	2	.73	1.24	n.s.
PMA	4	3.26	5.23	< .01
PTA	4	.49	1.04	n.s.
MTA	2	.34	.40	n.s.
PMG	2	.89	1.51	n.s.
PTG	2	.49	1.04	n.s.
MTG	1	.45	.52	n.s.
PAG	4	.92	1.07	n.s.
MAG	2	2.75	1.24	n.s.
TAG	2	.38	.39	n.s.
PMTA	4	.19	.50	n.s.
PMTG	2	.03	.08	n.s.
PMAG	4	.09	.15	n.s.
PTAG	4	.19	.40	n.s.
MTAG	2	1.96	2.28	n.s.

Table 18--Continued

Source	df	MS	F	p
PMTAG	4	.72	1.89	n.s.
S(AG)	54	10.09		
S(AG)M	54	2.22		
S(AG)P	108	.86		
S(AG)T	54	.97		
S(AG)MP	108	.59		
S(AG)PT	108	.47		
S(AG)MT	54	.86		
S(AG)MPT	108	.38		
TOTAL	719			

whereas the 180° was easiest the other positions in the rotation response mode ($p < .01$), proved more difficult than the others in the construction mode ($p < .01$). The response mode x item type interaction effect presented in Figure 22 discloses that while subjects perform better in the construction response mode when perspective-taking tasks involve Euclidean items, no differences between object types are found with the rotation response.

Figure 23 depicts the significant interaction of item type x age. Item type is not differentiated in the performances of four year olds but both six and eight year olds perform better when presented with Euclidean arrays.

The significant second-order interaction of position by response mode by age is represented in Figure 24. Four-, six-, and eight-year-old children are not significantly affected by response mode in their perspective taking at the 90° position, and responses to the 90° and 270° positions fail to significantly differ by age despite response mode. However, while performances by four versus six year olds and six versus eight year olds at the 270° position fail to reach significance in the construction response mode, four year olds do less well than six year olds at this position ($p < .05$) via the rotation response mode. Eight year olds differ from four year olds in both response modes ($p < .05$). The two response modes are best differentiated by performances of the various age groups at position 180°. In the construction response mode six year olds perform less capably than eight year olds ($p < .05$), while the four and six year old contrast fails to reach significance. The reverse is true for the rotation response mode in which four year olds do not perform as well as six year olds ($p < .01$), who do not significantly differ from eight-year-old subjects.

Furthermore, this second-order interaction also reveals that the performances of four year olds do not differ at any of the three positions with either response mode. In the construction response mode, eight year olds ($p < .01$) and six year olds ($p < .05$, $p < .01$, respectively) perform better at the 90° and 270° position, which fail to differ, than they do at the 180° position. However, while the performance of the six and eight year olds still fail to differ with the rotation response mode at the 90°

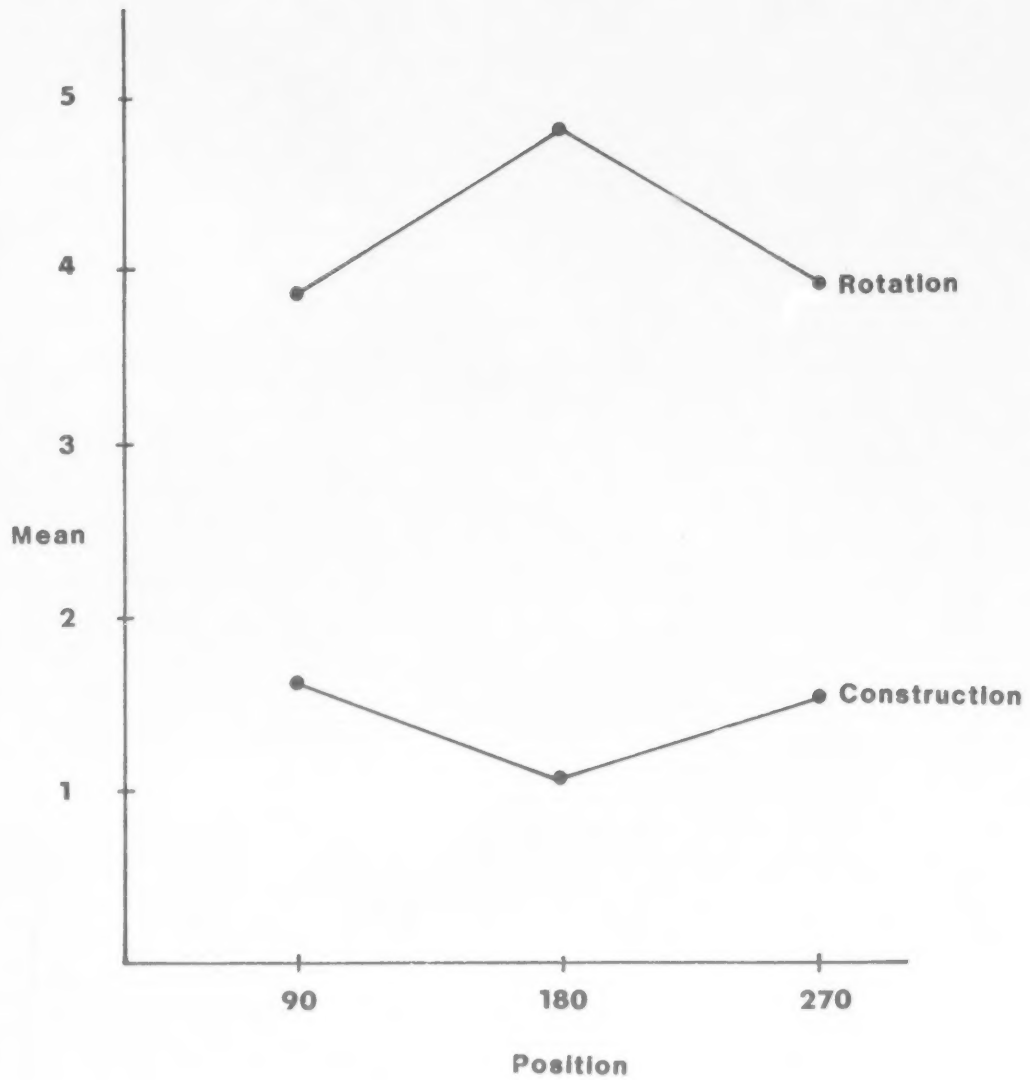


Figure 21: Mean number correct as a function of position x response mode. Study 6 data

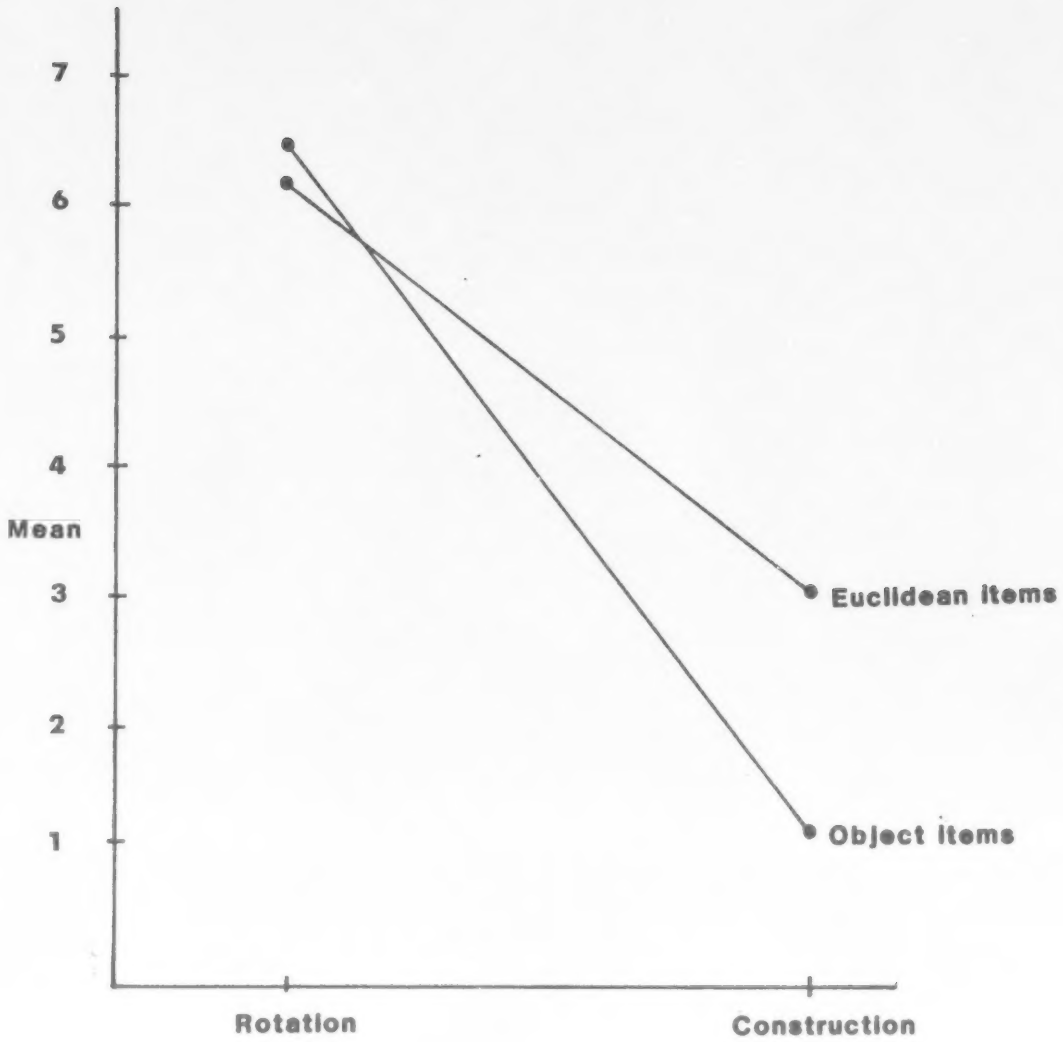


Figure 22: Mean number correct as a function of response mode x item type. Study 6 data

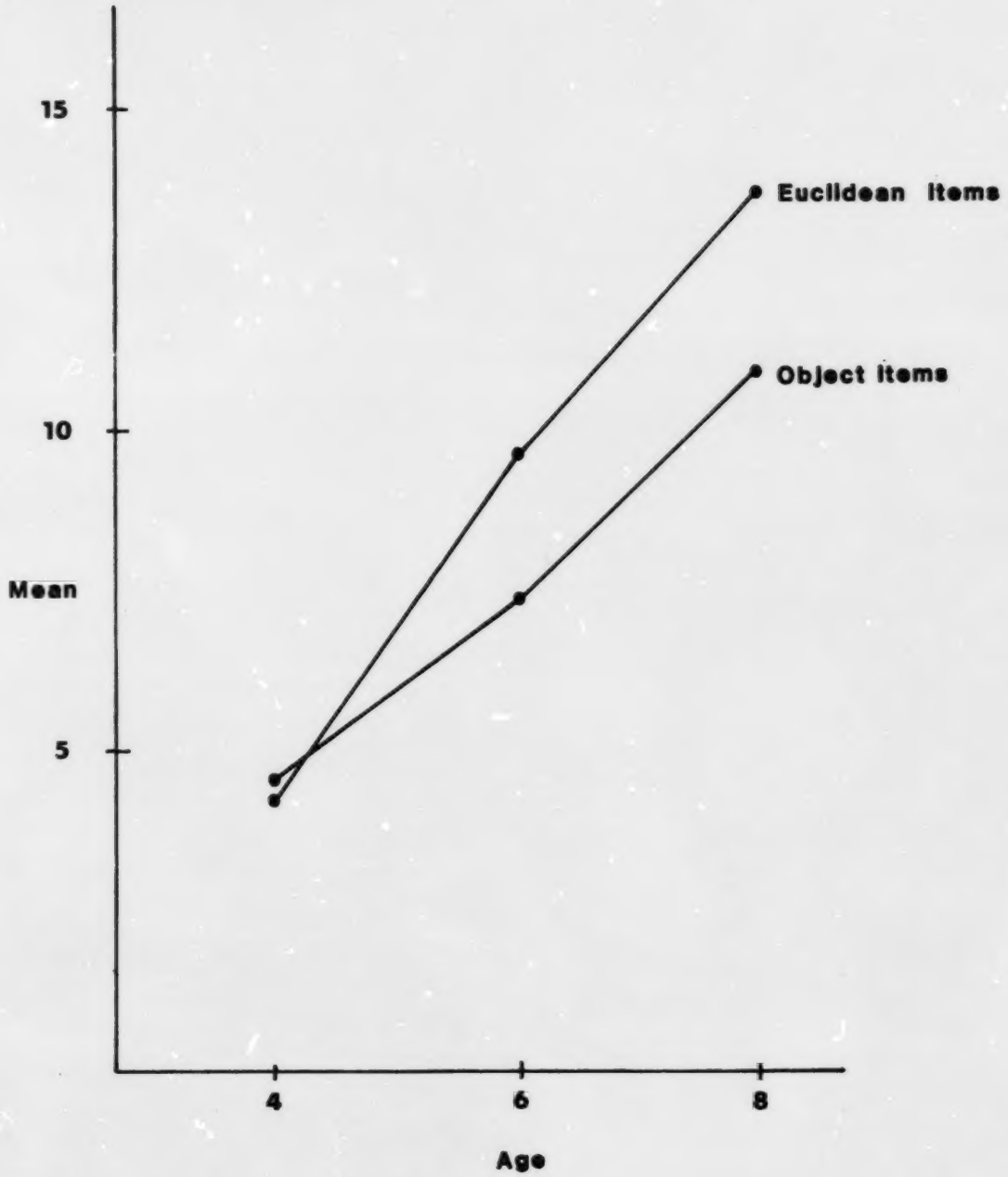


Figure 23: Mean number correct as a function of item type x age. Study 6 data

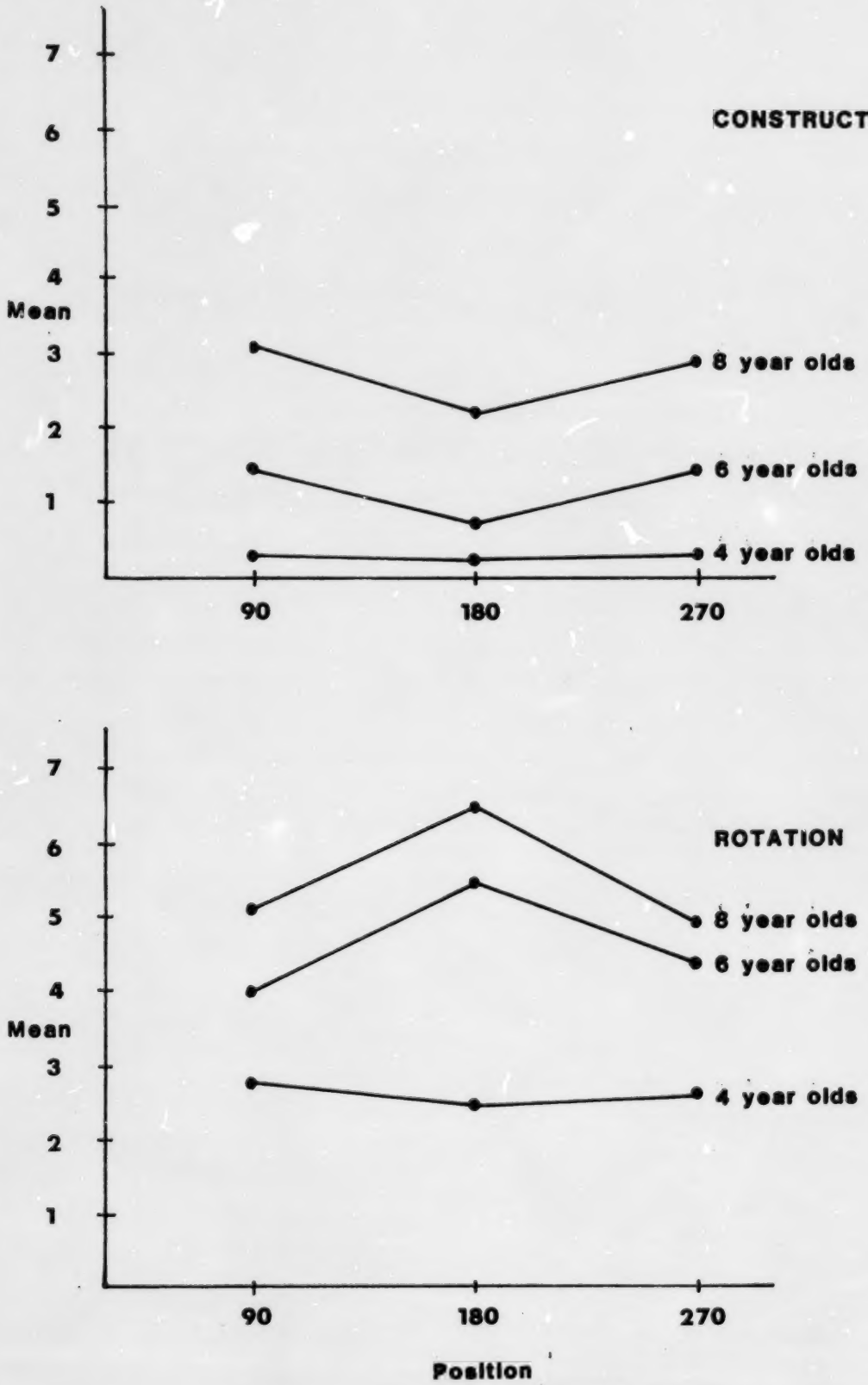


Figure 24: Mean number correct as a function of position x response mode x age . Study 6 data

and 270° positions, both age groups now perform better at the 180° position ($p < .01$).

4. Discussion

The results of this investigation fail to add much substantial information over those suggested by the previous studies' findings. The commonalities between the investigations serve as a replication of effects; the discrepancies are most probably due to experimental artifact rather than to any possible differences in psychological processes. The age effect of study 5 is reiterated in study 6, as is the relative ease of a rotation response over a construction one. These results are neither surprising nor unexpected.

The major discrepancies between studies 5 and 6 occur with item type and when item type interacts with other variables. In all likelihood this non-parallelness reflects non-parallelness in the selection of items utilized rather than any difference associated with dimensionality per se. The complexity of the Euclidean displays was identical across studies 5 and 6; this was not the case for non-Euclidean object displays. When presented two-dimensionally in study 5, the non-Euclidean array items, black and white line drawings of single objects, were more simplistic than the three-dimensional colored non-Euclidean materials employed in study 6. Correct performance in the former study would only necessitate that the subjects infer that the viewpoint at 180° was an upside-down-horse, car, or house. The two or three color Euclidean arrays in study 5 consisted of two to five items and were thus more complex than the non-Euclidean materials. In contrast, the study 6 three-dimensional non-Euclidean and Euclidean arrays both consisted of multiple item arrays. Thus, both Euclidean and non-Euclidean arrays in study 6 necessitated conservation of the internal relationships among items within the array while only the Euclidean items required this concept in study 5. The findings that non-Euclidean items were easier in study 5 and not easier in study 6 is consistent with stimulus complexity differences between the two studies.

Results of study 6 indicating that in fact non-Euclidean items were more difficult than Euclidean ones again can be attributed to stimulus complexity. Each item in the Euclidean display consisted of individual blocks placed together in an arrangement. While the entire arrangement

necessitated attention to left-right and up-down relationships, the individual pieces making up the arrangement were symmetrical without left-right, up-down differences. Individual pieces in the non-Euclidean arrays did have unique characteristics distinguishing left from right and front from back. In these latter items, therefore, two sets of spatial orientation features existed, i.e., those for the whole array and those for individual pieces. With a construction response, both sets would also have to be coordinated, while with a rotation response this would not be necessary. If the number of levels of left-right, front-back relationships is an important consideration in children's perspective taking, one would expect that: (1) Euclidean and non-Euclidean items would be equivalent in a rotation task and (2) non-Euclidean items would be more complex in a construction task. The empirical findings support these predictions.

With both stimulus and response demand differences in mind, the interpretation of findings with respect to position becomes more meaningful. For example, in the rotation mode the 180° position is easiest. If the child is only focusing on one aspect of the array and reversing it, solving the 180° perspective task eliminates the need for coordinating any left-right relationship; solving the 90° and 270° perspectives requires at least one left-right decision. This is supported empirically since in the rotation task for the six and eight year olds 180° was easier than either 90° or 270° the last two of which are equivalent; the four year olds' level of sophistication in the task was so poor that their responses can probably be attributed to chance.

On the other hand in the construction response, the 180° position could prove the most difficult. Both levels of right-left orientation would have to be coordinated and reversed. If left-right orientation is problematic for young children, this perspective judgment should pose the most problems. This prediction too is empirically supported.

In total, study 6 seems primarily to have reiterated study 5 and further points out the differences between a rotation and construction response. Rotation would appear to tap orientation of the whole only; construction requires both orientation of the whole plus orientation of the parts. Clearly the two tasks are not equivalent in terms of assessing the understanding of spatial relationships.

D. Study 7

1. Research Objective

While studies 5 and 6 answered questions concerning the perspective-taking capabilities of children when they work with two- and three-dimensional materials, the issue of cross-dimensionality had not yet been examined. Do young children see the identity between representations and the reality those representations depict? This issue is particularly relevant since we live in a three-dimensional world but generally teach children about that world via a two-dimensional media. The ability to comprehend and manipulate a three-dimensional object via its two-dimensional representation is a skill integral to architecture, engineering and drafting as well as many other technical fields. If we concern outselves with the development of spatial literacy, we must address the issue of cross-dimensionality with transformational tasks such as the coordination of perspectives.

A second issue which we felt should be considered is the importance of the level of abstraction in the utilized representations. In our western culture, we depend heavily on two types of two-dimensional representations, photographs and perspective drawings. We somehow have learned that these flat depictions capture a three-dimensional reality. While this behavior is more conceptual than perceptual, it is not taught in any type of systematic format. It is this matching of identity between a two-dimensional representation of a three-dimensional object which is the main concern of this study. However, this equivalence judgment may be differentially effected by the level of abstractness present in the representation. Perhaps the identity matching is developmentally easier when photographs are employed as opposed to more abstract perspective drawings. Both types of representations will be utilized in our dimension-crossing transformational tasks.

2. Method

a. Subjects. One hundred and eight of the 120 subjects in study 5 were included in this study. Subjects in each age group were randomly assigned to direction of transformation treatment conditions so that there were an equal number of each age and sex in the two treatment conditions.

b. Materials. Eight stimulus arrays of the types employed in study 6 were utilized. The first four arrays were composed of non-Euclidean objects while the last four were composed of Euclidean materials. The arrays were presented in a fixed order for all subjects. Three inch by 5 inch photographs were taken of the 0°, 90°, 180°, and 270° rotation positions of each array, for a total of 32 photographs. Tracings were made of the photographs and then colored and mounted on 4" x 6" cardboard pieces.

c. Scoring. Each perspective response was directly checked off in an appropriate box on the coding sheet which indicated which position the subject had selected. There was a possible total of 32 correct responses in the photograph condition with an additional 32 correct responses in the drawing condition. Thus, the total possible score, within a direction of transformation condition, was 64 points.

d. Procedure. In the first treatment condition (2DS, 3DR) subjects were presented with a two-dimensional stimulus and asked to respond via a three-dimensional response item. The presented stimulus items were either photographs or drawings of the four perspectives of each array. The order of presentation of stimulus type was counterbalanced. Half of the subjects from each age group were selected to be first presented with drawings of all eight arrays, followed by photographs of the same eight arrays or with photographs of all arrays preceding the presentation of drawings.

The position request order for all arrays was a randomly derived sequence which remained invariant for all subjects. Each time, E would place a perspective representation, either a drawing or photograph, horizontally on the table in front of the S. E sat directly to S's right. E would say "Point to the side which I took this photograph from" or "made this drawing from." All four positions for each array were measured before the next array was produced.

The second experimental condition (3DS, 2DR) involved the presentation of three-dimensional stimulus arrays. The same eight arrays were employed and presented in the same sequence as in the two-dimensional stimulus presentation. At the beginning of perspective-taking assessment with each array, E placed two-dimensional representations, either photographs or

drawings, of the array's four viewpoints in front of S. E moved to a rotation position and asked S to point to the photograph or drawing which showed what E was seeing. The representations were rearranged before each position change. The presentation order of representation type was counterbalanced. As in the two-dimensional stimulus, all of the eight arrays were first responded to by either photographs or drawings and were then responded to with the opposite representation type.

3. Results

The correct responses to the two experimental conditions were analyzed by means of a 3(age)x2(sex)x2(direction of transformation)x4)position x2(representation mode, either photographs or drawings)x2(item type) analysis of variance with repeated measurement of the last three factors. This analysis is summarized in Table 19. Significant main effects were found for age ($F_{2,95} = 21.38$, $p < .005$) and for position ($F_{3,289} = 59.54$, $p < .001$). Post hoc analyses revealed that eight year olds performed better than six year olds ($p < .01$) who performed better than four year olds ($p < .01$). The 90° and 270° positions failed to differ from one another but proved more difficult than the 180° position ($p < .01$) which was more difficult than the 0° position.

A first-order interaction ($F_{1,96} = 4.69$, $p < .05$) for item type by representation mode (Figure 25) revealed that photographs were more facilitative of perspective-taking performance with non-Euclidean materials than with Euclidean materials ($p < .01$). No significant difference was found between item types when the drawing representation mode was utilized.

A representation mode by sex interaction effect ($F_{1,96} = 4.58$, $p < .05$) represented in Figure 26 showed that males performed better than females with the photograph representation mode ($p < .01$) but they did not differ significantly in the drawing representation mode. The age by position interaction ($F_{6,288} = 5.77$, $p < .001$) displayed in Figure 27 discloses that four and six year olds exhibited the same response pattern to the four positions. The 0° position was easier ($p < .05$) than the other three positions which failed to differ. However, a different pattern is evident for eight year olds who found the 90° and 270° positions more difficult than the 180° position which was more difficult than the 0° position. Furthermore, the 0° position differentiated the three age groups with eight year

Table 19

Summary of Analysis of Variance: Study 7 Data

Source	df	MS	F	p
P (position)	3	142.90	59.54	< .001
T (item type)	1	3.25	3.28	ns
M (response mode)	1	.17	.17	ns
G (sex)	1	19.38	1.90	ns
A (age)	2	217.69	21.38	< .005
D (direction of transformation)	1	29.82	2.93	ns
PT	3	.45	.73	ns
PM	3	.39	.60	ns
TM	1	3.80	4.69	< .05
PG	3	1.26	.53	ns
TG	1	.63	.26	ns
MG	1	4.58	4.58	< .05
PA	6	13.84	5.77	< .001
TA	2	2.03	2.05	ns
MA	2	.70	.70	ns
GA	2	25.96	2.55	ns
PD	3	43.35	18.06	< .001
TD	1	.88	.88	ns
MD	1	.88	.88	ns
GD	1	13.90	1.37	ns
AD	2	14.29	1.40	ns
PTM	3	1.25	2.02	ns
PTG	3	.37	.60	ns
PMG	3	.45	.69	ns
TMG	1	.56	.69	ns
PTA	6	1.57	2.49	< .05
PMA	6	.76	1.17	ns
TMA	2	1.21	1.48	ns
PGA	6	3.32	1.38	ns
TGA	2	.67	.68	ns

Table 19--Continued

Source	df	MS	F	p
MGA	2	.39	.39	ns
PTD	3	.24	.39	ns
PMD	3	.42	.65	ns
TMD	1	.36	.44	ns
PGD	3	.03	.01	ns
TGD	1	1.63	1.65	ns
MGD	1	1.07	1.07	ns
PAD	6	5.39	2.25	< .05
TAD	2	2.03	2.05	ns
MAD	2	.55	.55	ns
GAD	2	10.64	1.05	ns
PTMG	3	.41	.85	ns
PTMA	6	.88	1.83	ns
PTGA	6	.72	1.16	ns
PMGA	6	.56	.86	ns
TMGA	2	1.37	1.69	ns
PTMD	3	1.89	3.94	< .01
PTGD	3	.22	.35	ns
PMGD	3	.08	.12	ns
TMGD	1	.10	.12	ns
PTAD	6	.55	.89	ns
PMAD	6	.30	.46	ns
TMAD	2	.25	.31	ns
PGAD	6	3.84	1.60	ns
TGAD	2	.60	.61	ns
MGAD	2	.04	.04	ns
PTMGA	6	.30	.63	ns
PTMGD	3	.25	.52	ns
PTMAD	6	.80	1.67	ns
PTGAD	6	.45	.73	ns
PMGAD	6	.22	.34	ns
TMGAD	2	.18	.22	ns

Table 19--Continued

Source	df	MS	F	p
PTMGAD	6	.30	.63	ns
S(AGD)	96	10.18		
S(AGD)P	288	2.40		
S(AGD)M	96	1.00		
S(AGD)T	96	.99		
S(AGD)PM	272	.65		
S(AGD)PT	288	.62		
S(AGD)TM	96	.81		
S(AGD)PTM	288	.48		

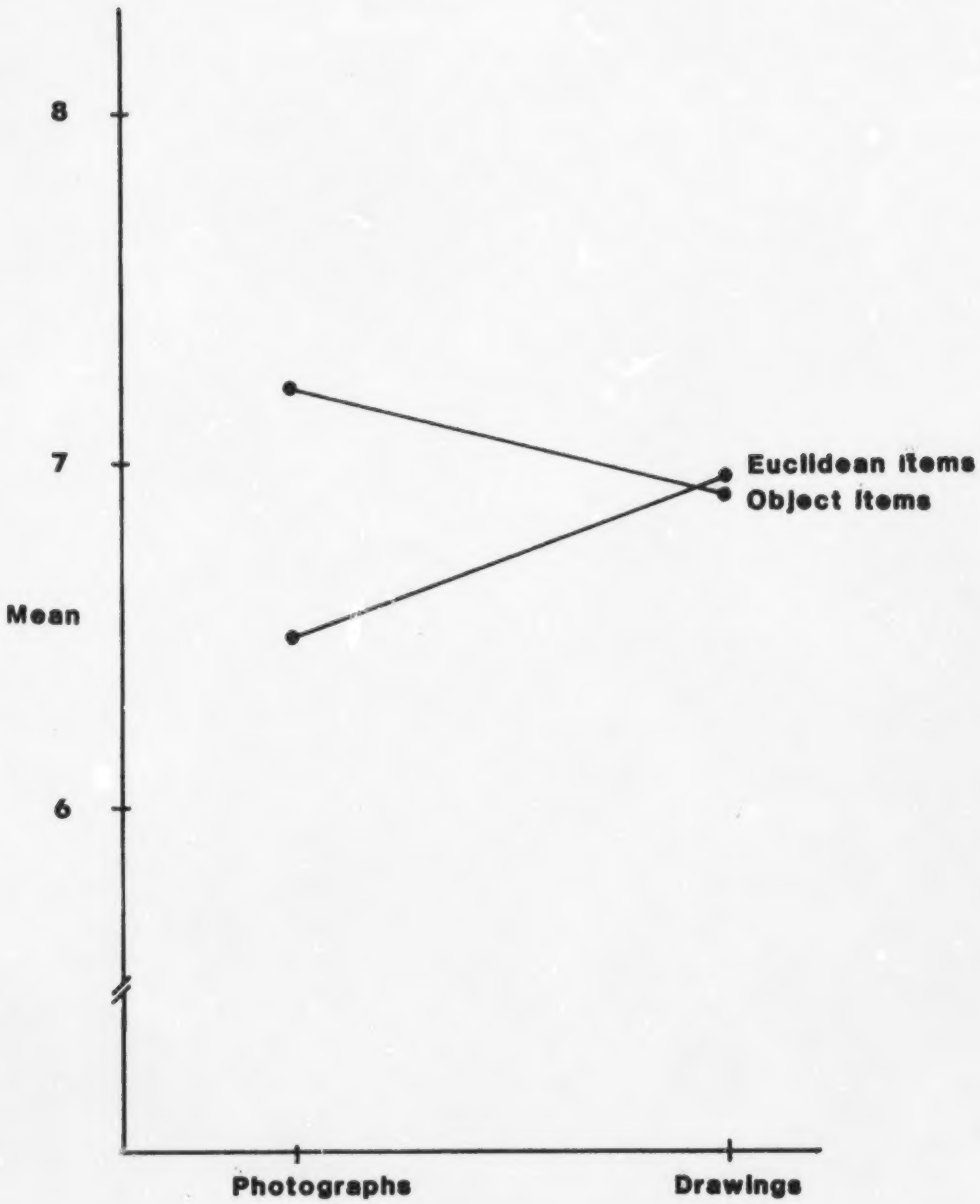


Figure 25: Mean number correct as a function of item type x representation mode . Study 7 data

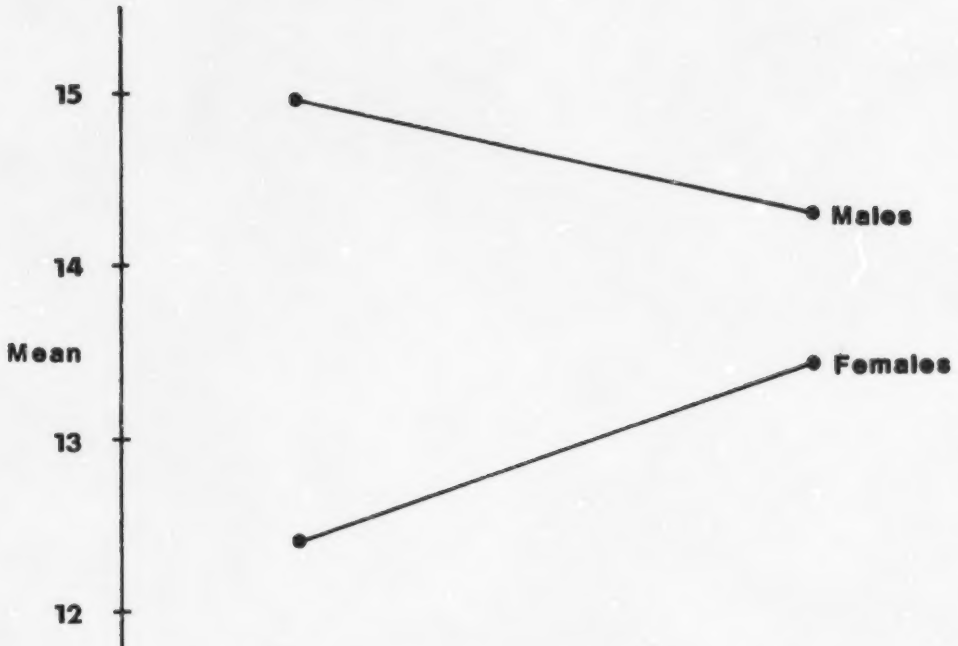


Figure 26: Mean number correct as a function of sex x response mode.Study 7 data

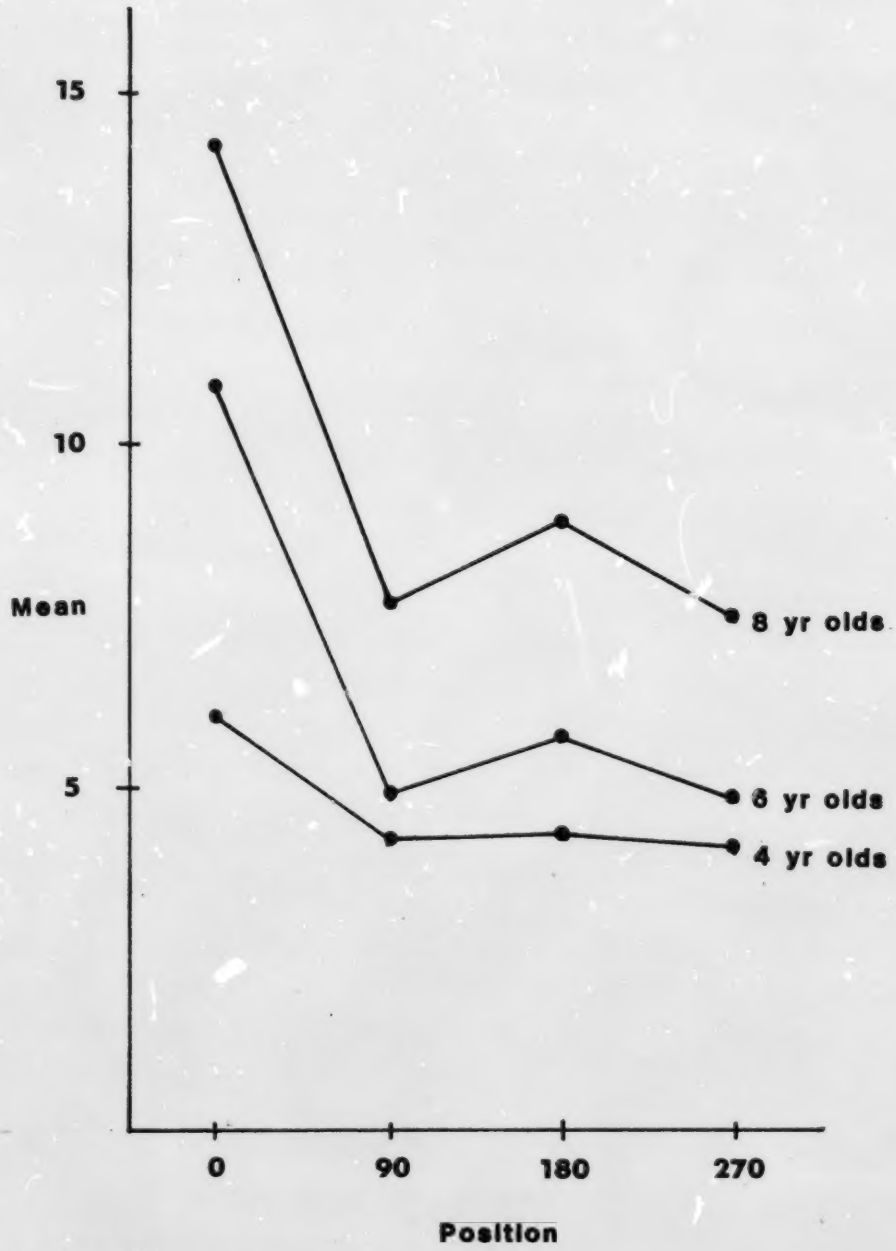


Figure 27: Mean number correct as a function of age x position. Study 7 data

olds outperforming six year olds ($p < .01$) who performed better than four year olds ($p < .01$). With the 90°, 180° and 270° positions, eight year olds performed better than six year olds ($p < .05$) who failed to differ significantly from the four year olds.

A position by direction of transformation interaction seen in Figure 28 ($F\ 3,288 = 18.06$, $p < .001$) indicates that when items were presented two-dimensionally and responded to three-dimensionally, the 0° position was easier than the 180° and 90° position ($p < .01$) which failed to significantly differ. The 180° position was easier than the 270° position ($p < .05$) which also did not differ from the 90° position. However, when stimulus items were presented three-dimensionally and responded to two-dimensionally the 0° position was still easier than the 180° position ($p < .01$) but both the 270° and 90° positions, which did not differ, proved more difficult than the 180° position ($p < .01$).

Furthermore, while the 2DS-3DR was easier at the 90°, 180° ($p < .01$) and 270° ($p < .05$) positions, 3DS-2DR was easier at the 0° position ($p < .01$).

A second-order interaction was found for age by position by item type ($F\ 5,288 = 2.49$, $p < .05$) and is depicted in Figure 29. The age contrasts show that whereas the 180° position differentiated the eight-year-old subjects from the four-year-old subjects with non-Euclidean materials ($p < .01$) no such differences were found between the age groups at the 180° position when Euclidean materials were employed. There were no significant differences between the age groups at the 90° and 270° position with either Euclidean or non-Euclidean material. The greatest age discrepancy between responses to the Euclidean and non-Euclidean materials came from the 0° position. With non-Euclidean objects eight year olds performed better than six year olds ($p < .05$) who performed better than four year olds ($p < .05$). However, with the Euclidean materials, eight year olds did not differ from six year olds.

In the position contrasts four and six year olds performed approximately the same at the four positions with both Euclidean and non-Euclidean materials. There were no significant differences among positions for the four year olds while the 0° position was easier than the other three positions for the six year olds ($p < .05$). The latter finding was also true for

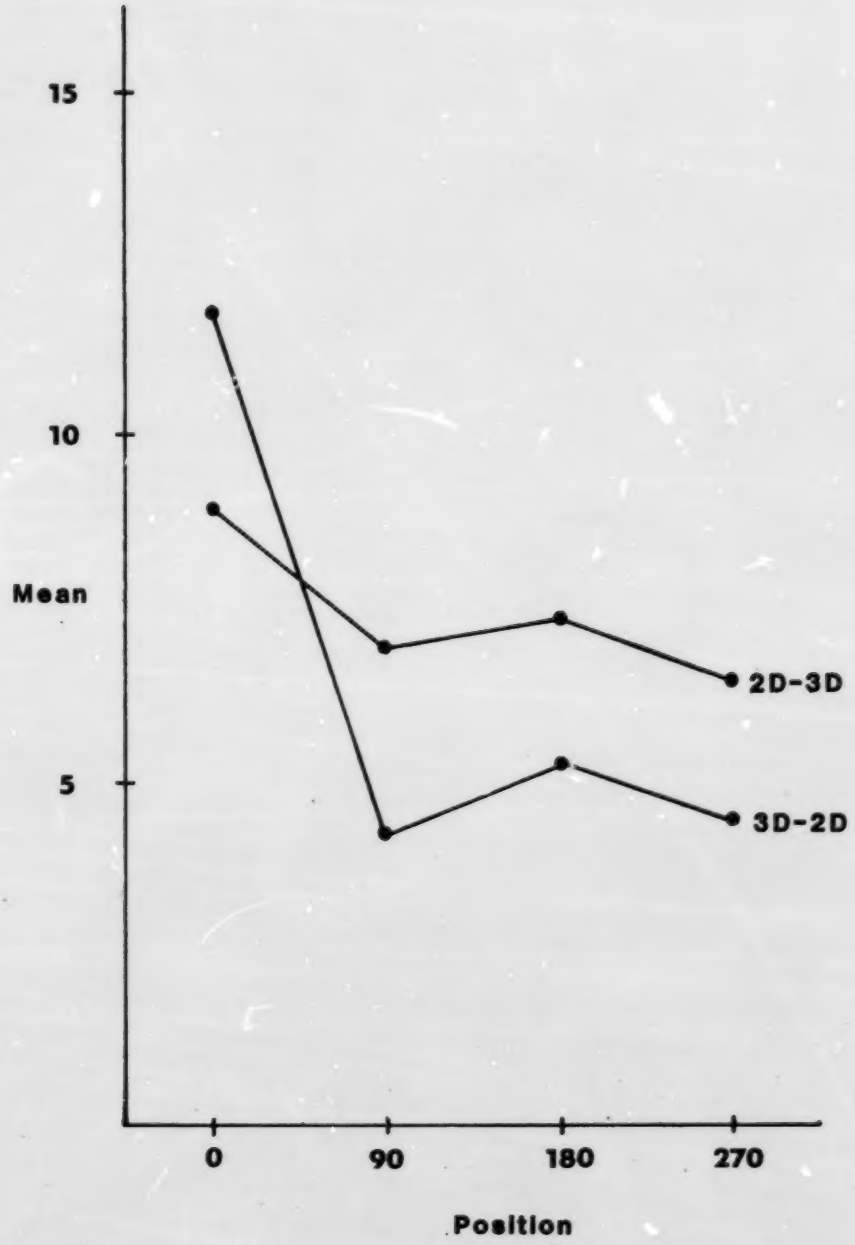


Figure 28: Mean number correct as a function of position x direction of transformation. Study 7 data

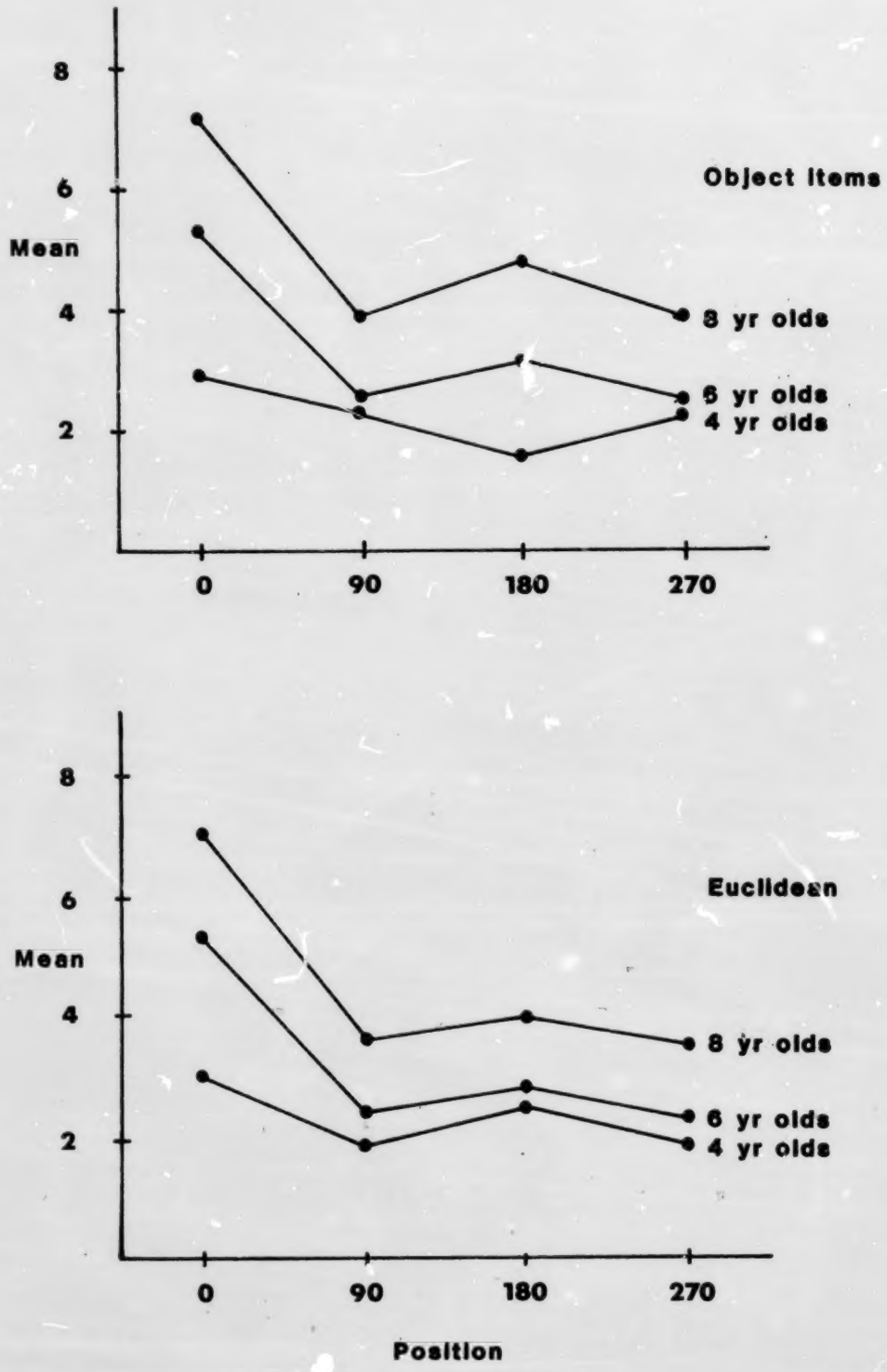


Figure 29: Mean number correct as a function of age x position x item type. Study 7 data

eight year olds responding with Euclidean materials while the 0° position was easier than the 180° position ($p < .01$) which was easier than the 90° and 270° positions ($p < .05$) with Euclidean materials. Positions 90° and 270° failed to differ.

The item type contrasts show that, for the three age groups, non-Euclidean materials do not differ significantly from Euclidean materials at the 0°, 90°, or 270° positions. However, non-Euclidean materials proved easier than Euclidean materials at the 180° position for eight year olds ($p < .01$) while the opposite effect was true for the four year olds ($p < .05$). No difference was found between Euclidean and non-Euclidean materials for the six year olds' performance at the 180° position.

Another second-order interaction was found for the age by direction of transformation by position factors ($F_{6,288} = 2.25$, $p < .05$) and is depicted in Figure 30.

At the 0° position, eight year olds perform better than six year olds ($p < .01$) who outperform four year olds ($p < .01$) in both the 3DS-2DR and 2DS-3DR conditions. Age contrasts for the 3DS-2DR condition failed to find any significant differences between the three age groups at the 90°, 180° and 270° positions. Eight-year-old subjects performed better than six year olds ($p < .01$) who did not differ significantly in performance from four year olds at the 90°, 180°, and 270° in the 2DS-3DR condition.

Direction of transformation contrasts for eight and six year olds at the 0° position revealed that performances were better in the 3DS-2DR condition. However, while there were no differences between the direction of transformation conditions for six year olds performing at the 90°, 180°, and 270° positions, eight year olds performed better ($p < .01$) in the 2DS-3DR with these same three positions. Four year olds did not differ across direction of transformation conditions at either the 0° or 180° positions while their performances were better in the 2DS-3DR condition at the 90° ($p < .01$) and 270° ($p < .05$) positions.

Analysis of the position contrasts in the 3DS-2DR condition revealed that all three age groups performed better at the 0° position than the 180° position ($p < .01$) which resulted in better performances than the 90° and 270° positions ($p < .01$). The 90° and 270° position performances of the two condition groups did not differ significantly.

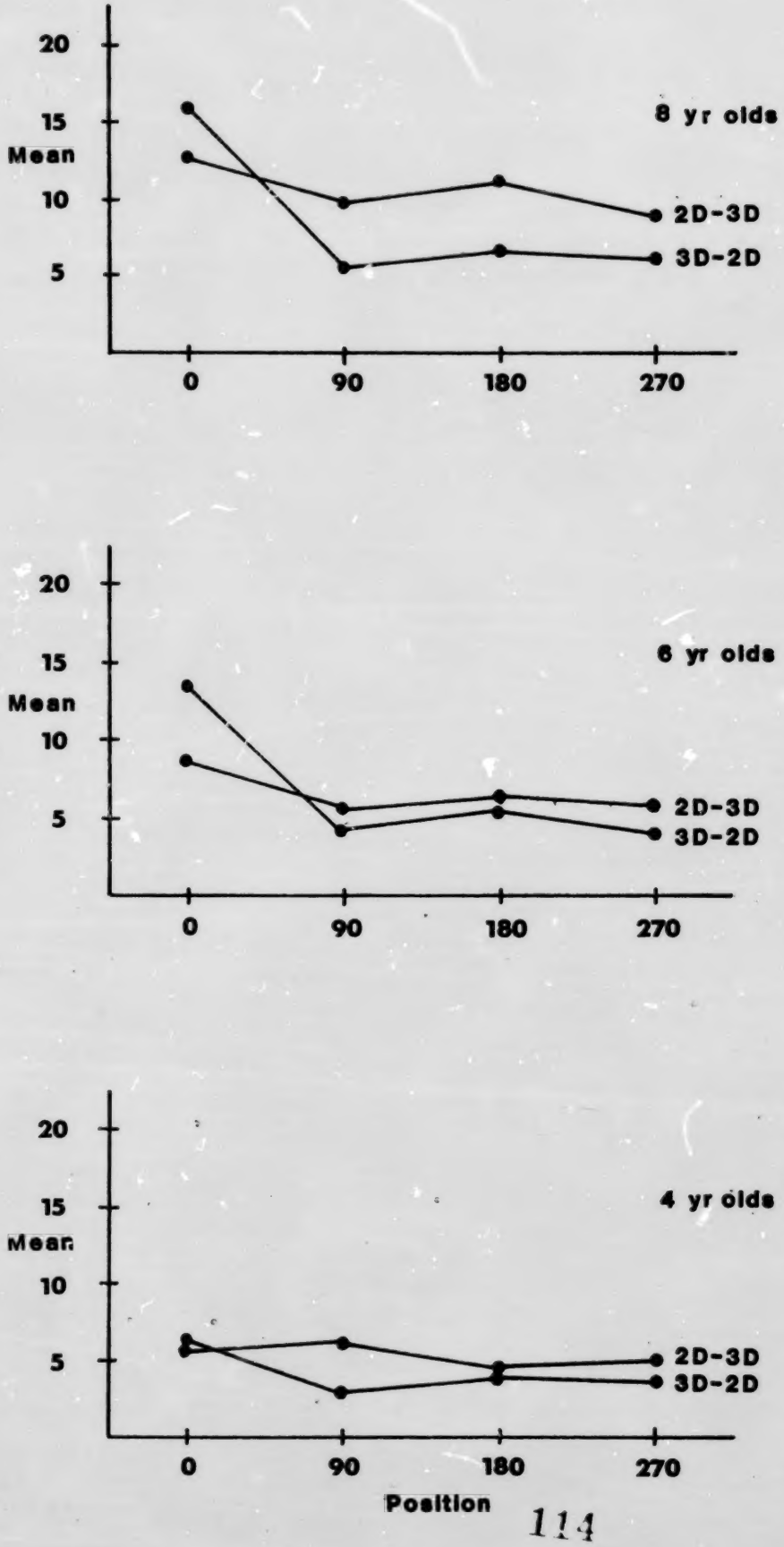


Figure 30: Mean number correct as a function of age x position x direction of transformation. Study 7 data

Performances across the three age groups do differ at various positions in the 2DS-3DR condition. Eight- and six-year-old subjects perform better at the 0° position than the 180° position ($p < .01$) which proves more facilitative of correct responding than the 90° position ($p < .01$). However, while eight year olds perform better at the 90° position than at the 270° position ($p < .01$), no significant differences were found between six year old's performances at these two positions.

4. Discussion

Interpretation of the empirical results of this study is an ambiguous task both in and of themselves and in contrast with the earlier investigations. The design was complex and powerful leading to a number of significant interactions which only exacerbate the difficulty of drawing conclusions. In addition, the selection response which probably facilitates considerable guessing further complicates the problem. With these difficulties in mind, therefore, we have chosen to focus only on the obvious trends in the data rather than risk overinterpreting the more subtle complexities.

First, while the age trend is reiterated here, the performance of four year olds is essentially at chance level for perspective judgments and only slightly above chance at the 0° position. Interpretation of a possible random response pattern would certainly seem unwise. That these young children lack sophistication is not particularly surprising in view of the stimulus complexity, which parallels study 6, and the necessity of crossing dimensions at the same time. More surprising is the low performance on the 0° position which is only a matching rather than a perspective-taking task. These youngsters perform only slightly above chance even with this low level task. It would seem that four year olds are not really able to recognize the correspondence between a 3-dimensional spatial representation and its 2-dimensional depiction; therefore whether they are able to solve perspective tasks under those conditions becomes a rather meaningless question.

Examining only the data for the six- and eight-year-old subjects, the results parallel those of studies 5 and 6. For example, the pattern of responses across positions again pinpoints the relative ease of the 180° position and the 0° position as compared with both the 90° and 270° view.

The means for the six year olds and eight year olds at 0° ($\bar{X}$ = 5.34 and 7.09, respectively out of a total possible of 8) suggest that these children are able to cross dimensions in at least the matching task. Means at the 180° are also well above chance for both groups and exceed those for the remaining two positions. This pattern of success across the three perspective positions reiterates the findings with the selection and rotation response in study 5 and the rotation response in study 6. This is not surprising. In the selection mode as utilized here, the subject need only attend to one object in the array and judge its relative position rather than coordinate internal spatial relationships. Hence the selection task, like the rotation response utilized previously, appears to be a level 1 problem. In our previous findings the 180° view is the easiest when this level of comprehension is assessed.

There is an additional limitation in this study that predisposes the authors to be cautious about drawing any further conclusions. The photographs and drawing representations in the 2-dimensional form were nearly identical; to propose that one was more abstract than the other is probably an overstatement. Therefore, interpretation of the sex effect interactions and item-type interactions is omitted.

E. General Conclusions in Perspective Taking

Perspective taking is an important spatial skill to examine in young children, but the assessment of it presents a variety of problems. Some of these problems relate to the mediation processes children utilize in solving these problems. Mental rotation of the array could be one means as could rotation of the self about the array. These means have been suggested by others (Huttenlocher & Presson, 1973) as possible mediational strategies. However, subjects could also utilize other strategies such as rotation of individual items, reconstruction or verbal means. It is difficult to determine which strategy an individual uses, whether a combination of strategies is used jointly, or whether strategy within the individual changes with external stimulus dimensions and response demands.

Logically it would seem that if purely mental rotation were a high frequency strategy, then the 180° position should not prove easier than either of the other two; it would have to be rotated further than the

two side positions and information could be lost. In fact, one would expect the side positions requiring less rotation would prove easier. This was not the case across these three empirical investigations. Similar outcomes could be predicted for rotation of the self about the array. Unfortunately these results reported here do not allow one to infer strategy.

Other problems with perspective-taking tasks involve response mode. As has already been indicated, different response modes most likely tap various levels of spatial understanding which are not equivalent. The rotation task and the selection task require much less sophistication than a construction task since the former may only require attention to one object or to the array as a whole rather than coordination of relations between objects within an array. Knowing that an object appears different from a multitude of positions is far less complex than coordinating a spatial relationship across a rigid motion transformation. The fact that young children can succeed at the former is not sufficient grounds for inferring comprehension of the latter. When assessment of spatial relationships is the focus, a rotational task would not seem to be the response mode of choice.

There are additional problems with the rotation response mode that could differentiate it from other tasks. For example, the rotation device provides the child with a motor analogue for problem solving that other response devices do not. Solution at a motor level and solution at a mental level are not necessarily of equivalent complexity nor are they necessarily isomorphic. Generalization, therefore, of results obtained with rotation devices should probably be suspect.

As with the first set of studies (1-4) we would contend that the construction response is the most sensitive. Correct responding on the basis of chance is minimal, perhaps infinitesimal. Errors analysis is possible, though yet to be conducted for these data, and understanding of internal spatial relationships is required. However, it would seem from the findings reported here that children younger than age six will experience marginal success.

V. The Examination of Transformational Abilities

A. Rationale for the Study

1. Introduction

The final study, like the first four studies in this series of research, investigates Lohman's visualization factor. However, whereas the first four studies required the reproduction of visual displays, this final study involved complex transformational tasks in an attempt to study the spatial processes utilized during the performance of spatial tasks.

The issue of relevant processes employed in spatial performances has been neglected both by American empiricists who tend to be performance based and avoid extensive inferential statements (Groen, 1978) and by the Genevans as a result of certain inherent limitations in the Piagetian approach. While Piagetian research has frequently resulted in developmental descriptions of specific concepts such as conservation, time, and others (Gruber & Voneche, 1978), these efforts have also been strongly influenced by a broader-based epistemological conviction that improvement in cognitive abilities results from ongoing interactions between the environment and hypothesized cognitive structures (Inhelder, Sinclair, & Bovet, 1974). This basic epistemological question has resulted in a research methodology built around the child's competence and patterns of thinking at a particular developmental level, and the contribution of environmental reality. A major limitation of such an approach to questions concerning cognitive processes is that while the Piagetian groupings provide a model of possible reasonings which children may use in performing spatial tasks, the actual spatial processes utilized are only minimally studied.

Factor analytic research, while not directly investigating psychological processes employed in problem solving, generally assumes that the shared variance accounted for by a particular factor is external indication of certain psychological processes common across subjects. According to this conception, individuals solving spatial visualization tasks should all be utilizing related cognitive processes. However, Lohman's (1979a, 1979b) work questions this assumption and instead suggests that identified factors primarily serve as gross indicators that specific tasks have certain common processing characteristics which may very well differ for various subpopulations.

A research question of particular significance, then, appears to involve the investigation of particular aspects of spatial processing (Montangero & Smock, 1976) in different subpopulations. The research to be described investigated two means of providing spatial information and how that information may be employed in spatial problem solving.

2. Review of Investigations into Spatial Processing

Although it remains a small subset of the general spatial concepts research, a few investigators have focused their attention on possible information-processing strategies which accompany spatial performances. Huttenlocher and Presson (1973) attempted to illuminate the cognitive processes which subjects may employ in spatial perspective taking. They compared the ability of third and fifth grade children to indicate the appearance of an array after an imaginary rotation on a single axis with children's prediction of how an array would appear to an observer at a different perspective. They found the rotation task to be easier than perspective taking for their subjects, thus providing confirmation for Lohman's later contention (1979a) that spatial relations and spatial orientation are separate factors. Huttenlocher and Presson (1973) conclude that subjects employ mental imagery as a mediator between initial visualization of the array and performance of the rotation and perspective prediction of that array.

Other investigators have also postulated the role of internal mediation in spatial tasks. Shepard and Chipman (1970) examined the isomorphism of internal representation retrieved from long-term memory to direct visual perception of the corresponding external objects and the relations among them. Judgments of shape similarity were collected from seven graduate students for all possible pairs (105) of 15 states under two sequentially presented conditions. In the first condition subjects were presented with only the names of the two states printed side-by-side on cards. Following this condition, subjects ranked adjacent pictorial depictions of each pair by shape similarity. Multidimensional scaling procedures were conducted and the results demonstrate that even when only the state names were given, states were judged to be most similar if their actual cartographic shapes are similar. They were not judged similar by

other possible associations such as acoustic features of the names or locations of the states. On the basis of their data, Shepard and Chipman (1970) conclude that the internal representations were more isomorphic to unpresented shapes than to explicitly presented names.

Shepard and Metzler (1971) further investigated the role of internal representation by measuring the response latency of college students required to recognize that two perspective drawings of geometric forms were either the same or different. The degree of angular separation between the two views forming a pair varied through a range of 0° to 180° by 20° increments about the vertical axis for orientation in depth and with the same range around the line of sight for orientation in the picture-plane pairs. Reaction time for the "same" pairs increased according to a linear function of angular difference in portrayed orientation, ranging from approximately one second at 0° to between four and five seconds at 180°. The provided interpretation of these results (Shepard & Metzler, 1971) suggests that shape comparison was accomplished by the rotation to congruence of a three-dimensional internal representation of one of the presented two-dimensional form drawings. These results were replicated in a subsequent experiment (Metzler & Shepard, 1974) with a similar interpretation.

Adopting the strategy employed by Shepard and Metzler (1971), Marmor (1975) tested the ability of five and eight year olds to determine equivalence or non-equivalence of two panda-bear-like shapes, differing in orientation on a range from 0° through 180° rotation. Reaction time was measured and increased linearly as a function of rotation degree in both age groups, suggesting that rotation of a mental image may be the process by which adult (Shepard & Metzler, 1971), eight- and five-year-old subjects perform the equivalence judgment task (Marmor, 1975).

In still another example of the reaction time functions in mental rotation studies, Cooper and Podgorny (1976) investigated the ability of adult subjects to discriminate previously learned standard versions of angular shapes from random distractor versions. Subjects were told a priori the identity and orientation of the test form. They were instructed to prepare for the presentation of the test form by mentally rotating in a clockwise direction an internal representation of the appropriate standard form

to the designated orientation. Subjects indicated that they had completed the rotation and were prepared for the test form. The time needed to prepare for the presentation increased linearly as the angular departure of the indicated orientation from the standard position was increased. Once again, these findings suggest that the subjects performed a mental rotation to prepare for the discrimination task, in accordance with their instructions.

A similar study (Shepard & Feng, 1972) was conducted with a task requiring more complex spatial manipulations, which if actually performed physically would require the folding up of an "unfolded" cube. However, the college-age subjects were not permitted to perform any physical manipulations on the stimulus items. The subjects were asked to determine whether two arrows placed on different folds of an unfolded cube would meet if the cube was folded up. An increasing linear relationship was found between response latency and the number of folds, as well as, the number of relevant squares carried with the folds, if the task were performed physically. A mental analog of a physical folding-up process was postulated as the most probable information-processing strategy adopted by subjects in their performances (Shepard & Feng, 1972).

Metzler and Shepard (1974) elaborate on the nature of this analog, describing the internal representation as embodying important structural features of the three-dimensional object, relative to a particular viewpoint, that would not be present in a two-dimensional projection of that object. Furthermore, they state that those features unique to the two-dimensional projection, with the exclusion of the particular perspective of the viewed object, have little influence on the internal process. However, Metzler and Shepard (1974) do not specify that this internal representation is a mental image, rather stating that some introspective reports suggest that the internal representation may not be purely visual. Thus, the representation may be a realtively abstract scheme which, while isomorphic to the spatial structure of the external object, is not tied exclusively to any specific modality.

Kosslyn (1978) adopts a stronger position in his model for mental imagery. He suggests that there is a quasi-pictorial spatial representation that is generated from more abstract representations. The qualifi-

cation implied by the term "quasi-pictorial" means that while the representation shares some properties of a picture such as spatial extent, brightness and texture, it does not necessarily have all the properties characteristic of the object itself.

Kosslyn (1978) argues that this spatial image is functional; that the experienced image is what the process of imagery is all about. According to Kosslyn (1978), an image is more like the evoked product of looking at a picture than the product engendered by a description of the picture. Evidence that the image has spatial characteristics and functions systematically in cognitive processing is drawn from a number of empirical investigations. Kosslyn, Ball, and Reiser (1978) found that more time was required to scan greater distances across visual images. More time was also required to see properties of subjectively smaller images than parts of larger ones (Kosslyn, 1975). Spatial boundaries of mental images which can interfere with the inspection of an entire image were discovered by Kosslyn in a study described in Kosslyn and Schwartz (1977).

Kosslyn (1978) cogently argues that the spatial image is probably generated from an underlying representation as opposed to being stored intact in long-term memory. He cites the greater difficulty in generating complex images, the fact that smaller images with fewer details are generated more quickly than subjectively larger images, and the fading of imagery over time, as evidence for his constructive image hypothesis. Two types of representations are purported to be used in image generation (Kosslyn, 1978). First, perceptual information or memories of the actual appearance of objects are employed in constructing images. Non-perceptual information such as that contained in verbal descriptions or abstract propositions could also be used in conjunction with the perceptual information. The necessity for including non-percpetual information is evident from subjects' abilities to construct novel images, as well as from introspective observations that the relations between parts of images may be represented in a more abstract fashion than by precise perceptual coordinates. Certain situations would be more amenable to an emphasis on such non-perceptual information. For example, in some cases, verbal descriptions may be more economical than a pictorial representation.

Based on this evidence, it seems very unlikely that images are stored in memory either in totality or in separate pieces. On the contrary, it appears much more likely that stored abstract information is used in conjunction with the literal appearance to generate imagery.

Kosslyn (1978) hypohtesizes a sophisticated processing system for imagery which assumes that the surface representation or images are generated from abstract deep representations stored in long-term memory. These deep representations would include stored propositional facts about the image such as where and how a given part is attached to the global image, how to locate that part as well as the optimal resolution necessary to see that part. This deep representation would enable subjects to generate images of specified sizes instead of generating a standard size image and then altering it to meet certain specifications.

Kosslyn asserts that imagery is used only if the sought information is not explicitly entered in the propositional "files." Kosslyn, Murphy, Bemesderfer, and Feinstein (1977) found that when items were blocked by their potential for being solved by semantic category information only and by those items necessitating imagery mediation, subjects would only use imagery when necessary. Otherwise, they solved the task by utilizing the category information.

The argument for the functional nature of the image is based upon the hypothesized existence of some type of processor which deals with the image in a manner similar to the visual perception processor's handling of sensory information. Kosslyn (1978) points to the parsimony of positing that the same processes would be used in both classifying surface representations produced in visual processing and in classifying those generated from memory. While the work of Shepard and his colleagues would be relevant in disucssing these shared processes, agreement has not been reached as to whether mental images, rather than some other form of internal representation, were operated upon in that series of reaction time experiments (Shepard & Metzler, 1971; Shepard & Feng, 1972; Metzler & Shepard, 1974; Cooper & Podgorny, 1976; Kosslyn, 1978). However, Kosslyn and Schwartz (1977) found that image transformations do occur more or less continuously and pass through successive intermediate stages, causing them

to point to the spatial characteristics of the image. Sharing of the same processing mechanisms would also account for findings reported by Kosslyn and Pomerantz (1977) that imagery selectively disrupts perception in the same modality.

Carrying on this pursuit of common internal processors, Finke and Kosslyn (1980) reasoned that if there were shared processors, properties of those structures that constrained perceptual acuity should similarly constrain mental imagery. Their findings reveal that just as acuity in perception is reduced as objects are observed at progressively more peripheral regions of the visual field, in the same way mental imagery acuity is reduced as objects are imagined at progressively more peripheral regions of the visual field. More dramatically, they suggest that perhaps the neural mechanisms that are activated when objects are imagined at specific locations are like those activated when objects are observed at particular locations in the visual field. Since there is a known neural basis in the size of receptive fields in the visual cortex, Finke and Kosslyn (1980) suggest that their results provide evidence for neural constraints limiting acuity in mental imagery.

3. The Role of Imagery in Spatial Problem Solving

The literature reviewed indicates that spatial imagery is viewed by many researchers as the mechanism for success on spatial tasks (Huttenlocher & Presson, 1973; Kosslyn, 1978; Marmor, 1975; Piaget & Inhelder, 1956, 1971). Being able to manipulate imagery has been presented as a defining property of a stage of spatial development characterized by sensitivity to the projective properties of space (Smothergill, Hughes, Timmons, & Hutko, 1975). Such imaginal processing is consistent with Piaget and Inhelder's work (1956, 1971). According to a Piagetian perspective, the adult's spatial conceptual system consists of sophisticated ideas about space and spatial relations (Piaget & Inhelder, 1956) which must be sequentially evolved from the naive, subjective notions of the young child. This evolution does not occur as a result of simple, direct perceptual experience where spatial knowledge merely reflects a copy of prior experience (Gruber & Voneche, 1978; Kosslyn, 1978). Rather, the child constructs spatial concepts through the organization of physical actions performed on objects in space. The

outcome is a representational spatial system that is conceptual and symbolic and thus different from simple perceptions, a description strikingly similar to the imagery description given by Kosslyn (1978). According to this conceptual imagery view, the image mediator employed in spatial problem solving can be a pictorial predecessor of an action not yet performed, an elaborate anticipation derived from both what is presently perceived and stored propositional facts about the perceived object or objects. When this type of dynamic imagery becomes purely intellectual, associated with operations and therefore transcends the bounds of sense perception, fully elaborated spatial concepts have been acquired (Piaget & Inhelder, 1956).

Furthermore, Piaget points out the intuitive match between space conceptualization and the mental image (Gruber & Voneche, 1978). There are additional consistencies: (1) both mental images and spatial concepts are conceived of as evolving from primitive perceptual notions to sophisticated conceptual ones; (2) both are presented as active constructions of the child surpassing the primarily passive perceptions which result from each individual centration, visual or tactile, which the child may make on an object; (3) and both derive from the child's motoric activity which coordinates these centrations of actual object perceptions. Thus, between perception and representation, Piagetians would argue that there occurs a whole series of increasingly systematized actions, internalized in the form of images, which culminate in mental operations (Piaget & Inhelder, 1956, 1971).

To summarize, the assumption of the facilitative function of imaginal processing upon performance of spatial tasks receives support from a variety of areas. Piagetian developmental theory insists upon its significance (Piaget & Inhelder, 1956, 1971), anecdotal reports indicate that many adults believe they employ image mediators while engaging in spatial problem solving (Shepard & Chipman, 1970; Kosslyn, 1978), and empirical evidence from a number of studies (Huttenlocher & Presson, 1973; Kosslyn, 1973; Marmor, 1975; Kosslyn & Schwartz, 1977; Kosslyn, Murphy, Bemesderfer, & Feinstein, 1977; Kosslyn, Ball, & Reiser, 1978; Pinker & Kosslyn, 1978; Finke & Kosslyn, 1980) is parsimoniously accounted for by spatial imagery processing.

However, the prominent cognitive processing role awarded to imagery by Piaget and Inhelder (1971) and other researchers (Huttenlocher & Presson, 1973; Marmor, 1975; Kosslyn, 1978) is not without dispute. Propositional theorists (Anderson & Bower, 1973; Chase & Clark, 1972; Pylyshyn, 1973; Reed, 1974; Anderson, 1978) have dichotomized the debate over the role played by imagery and state that the issue is not whether imagery exists, but what mental representations underlie imagery. Anderson (1978) purports that Kosslyn and his fellow researchers view imagery as being encoded in terms of spatial, modality specific properties, with a distinct code reserved for verbal information. Anderson (1978) and Pylyshyn (1979) claim that both imagery and verbal information is encoded in an abstract propositional format. The debate continues (Kosslyn, 1978; Pylyshyn, 1979) with no resolution in sight (Anderson, 1978). Meanwhile, how imagery may be differentially generated has received little attention. This research compares direct access to spatial imagery through a visual display of the relevant cues with indirect access to spatial imagery as a function of information delivered through verbal cues.

4. Verbal versus Imaginal Processing in Conceptual Tasks

Much of the research comparing verbal versus visual presentations has been conducted in the area of prose learning. However, these studies generally do not involve the kind of direct comparison between verbal and visual presentations described above. Rather, these studies emphasize the distinction between verbal and imaginal processing of information presented visually.

This body of research has demonstrated that children can improve their prose retention if they are instructed to make up internal images as a memory aid (Rasco, Tennyson, & Boutwell, 1975; Pressley, 1976). There is also evidence for a developmental trend between imposed imagery (experimenter provided pictures) and induced imagery (self-generated imagery). Kindergarten children demonstrate improved prose retention for stories read aloud to them if they are provided actual pictures depicting the content of each sentence while instructions to make up pictures in their head will not facilitate performance (Guttman, Levin, & Pressley, 1977; Ruch & Levin, 1977; Lesgold, Levin, Shimron, & Guttman, 1975; Shimron, 1975). It is not until approximately age eight that children's retention for heard

prose is improved by instructions to make up mental images depicting the story content (Guttman, et al., 1977; Shimron, 1975).

However, differential effects of imagery strategies versus meaningful verbal strategies have not been demonstrated. Typically, when imagery strategies have been compared with verbal ones (Levin, Bender, & Lesgold, 1976; Ruch & Levin, 1977) they have not been strategies of equivalent level. A spatial information-processing strategy which purported to compare verbal processing with visual processing in an object recognition and relocation within a spatial array task (Pezdek & Evans, 1979) also suffers from non-equivalent strategies. Subjects in both conditions were presented with a visual array with the additional attachment of a label to the objects in the verbal processing condition. Thus, in the visual group, the subjects had only the benefit of visual, perceptual information, while verbal condition subjects had perceptual information as well as semantic information about the array.

An important issue yet to be addressed is the comparative facilitation of information presented visually with information with information presented in an exclusively verbal format upon imaginal mediation in a spatial task. It is hypothesized that a visual presentation of spatial cues will provide a direct access to the generation of spatial imagery while a verbal presentation will result in an indirect approach to imagery formation. Two additional, and theoretically important, issues are whether children of different sex and age will be differentially successful depending upon whether the access to spatial imagery is direct or indirect.

B. Method

1. Subjects

One hundred and sixty predominatly Anglo, middle-class children from two public and two private schools comprised the sample group. Eighty of these children were enrolled in the second grade ($\bar{X}$ age = 8 years 1 mo) with the remaining 80 subjects being fifth graders ($\bar{X}$ age = 11 years 3 mos). In each grade level there were 40 males and 40 females. The subject sample was divided into four experimental groups with an equivalent number of subjects of each grade and sex in each group.

2. General Procedures

All subjects were seen individually by the experimenter for one session of approximately 15 minutes. The first step in the experimental procedure involved participation in a pre-measure which assessed subjects' knowledge of four elementary shapes: Square, circle, triangle, and rectangle. These shapes were embodied in foils and correct responses for subsequent tasks and it was deemed important for subjects to be familiar with these simple geometric forms.

There were two types of pre-measures. In the Drawing pre-measure subjects were shown four cards presented in a fixed order, each of which contained a representation of one of the four relevant shapes. Segments of these shapes were spatially separated and subjects were asked to draw a picture of what would result if the segments "were all connected." After each drawing, the experimenters produced another card depicting the correct representation. The experimenter either confirmed the subject's drawing by stating "Yes, that's what it would look like. What is it called?" while pointing to the prepared representation or provided corrective feedback, "No" or "That's not quite right. This is how it would look" again pointing to the prepared representation.

In the Labeling pre-measure, subjects were again exposed to the same four elementary shapes presented in the same order, but this time the line segments were connected. Each subject saw each shape twice in two different proportions, one of which was a match to the proportions in the connected shapes in the Drawing condition. Subjects were asked what each shape was called and were provided with confirmation, "Yes, it's a ____" or corrective feedback, "No, this is a ____." Time in the two pre-measure conditions was approximately equivalent.

Following the pretraining, subjects' spatial abilities were assessed between two different conditions with investigator-produced materials. All subjects performed on five trials with the same five geometric forms. Twenty-five responses were thus collected for each subject. In the course of the five trials, Ss were asked to indicate the appearance of (1) the bottom of the forms, (2) a horizontal section through the forms, (3) a vertical section through the forms, (4) a double horizontal section, and (5) the form "opened up" and laid flat.

3. Materials

The three-dimensional geometric forms were presented in the following fixed order for all subjects in both conditions: (1) the cylinder, (2) the rectangle, (3) the cone, (4) the house, and (5) the pyramid. However, trials order was varied for each subject on each shape in predetermined fashion. One hundred and twenty orders of response trials were possible; subjects were randomly assigned to one of those orders.

Responses were collected on each trial with a selection task in which the subject responded by pointing to one picture on a response board containing six possibilities. There were a total of 25 response boards. Each response board for each trial contained the correct response and five foils where the type of foils were consistent across items. One foil depicted a slice or an unfolding action taken on an equivalent two-dimensional perspective representation of the three-dimensional object. A second foil involved the outcome of this slicing action on a two-dimensional nonperspective representation. A third foil was a totally erroneous response. These three foil options were represented on the response boards for all five trials on each of the five forms. The last two foils varied somewhat depending on the trial. For the bottoms, horizontal sections, vertical sections and double horizontal sections, the fourth foil involved a confusion of the vertical and horizontal sections, while the fourth foil for the unfolding trial was a failure to account for all sides. The fifth foil for the bottoms, horizontal sections and vertical sections trials was a two-dimensional depiction of the object which indicated a failure to perform the operation, while for the unfolding it was a failure to understand proportionality, i.e., that the object dimensions were not constant throughout. Placement of the correct response and foils was randomzied across the six possible positions for each response board.

4. Scoring

Subject performance on each response for each trial and geometric shape was scored correct or incorrect. Possible scoring range across the five trials for five geometric shapes was 0-25 raw score points. For each trial, that range was 0-5. Tallies of the type of incorrect responses for each subject by each trial were also collected for errors analyses.

5. Experimental Conditions

a. Spatial cues visually presented. This group was shown a three-dimensional depiction of each of the geometric shapes for 10 seconds. The shapes were removed from the subjects' fields of vision before they were asked to commence responding to the trials. E sat in a position directly across from the child to maintain subject's attention and to facilitate control of exposure time. E presented a three-dimensional cardboard depiction of a cube saying, "Look at this cube." E looked at the shape for 10 seconds, removed the cube, and said, "I can do a lot of different things with that shape. First, I can figure out what the bottom of it looks like." E brought out a card with the cube bottom correctly drawn and laid it on the table directly in front of the child. E said, "This is what the bottom of it looks like. Here's something else I can do. I can take a slice out of it--just like a slice of bread. First, I can take a slice like this" (E made a horizontal slash from right to left in the air between the child and cube.) "and lay it flat on the table. See this is what that slice would look like. Also I can take a slice like this" (E made a slash from top to bottom in the air between the child and the cube.) "and lay it flat on the table." E showed child a card with a drawing of the vertical section of the cube saying, "That slice would look like this. Now I can figure out what I would get if I took out two slices like this" (E made one horizontal slash in air 1/2 inch from the top of the cube and another horizontal slash in air 1/2 inch from the bottom of the cube.) "and lay them flat on the table." E showed child line drawing of the double horizontal sections saying, "These slices would look like this. Finally, I can figure out how this cube would look if I opened it up and laid it flat on the table." E gestured to indicate an unfolding action. E showed child a correct drawing and said, "This is what that would look like." E presented this explanation of what was expected in the five trials only once, in the above fixed order and with a solid form (i.e., the cube) that was not used during assessment.

Following these trials explanation with the cube, E brought out the cylinder. Trials order was now randomly selected for each child. For example, E might have begun with the vertical sections trials. E would have said, "Look at this cylinder" and allowed 10 seconds to pass before

removing the cylinder. "You can do a lot of different things with that shape. First, I want you to figure out what you would get if you cut a thin slice out of the middle of that shape like this" (E made slash from top to bottom in the air between the child and cylinder.) "and lay it flat on the table." E presented the appropriate response board and said, "Now you point to show what that would look like." E allowed the child time to respond but repeated instructions if more than 20 seconds passed without a response. Following the child's response, E said, "Okay" after each trial within a shape and delivered praise "You're doing really well" after one solid had been removed from the table and before another solid was brought out. This non-contingent reinforcement was meant to maintain attention during a difficult task.

b. Spatial cues verbally presented. This condition was identical to the spatial cues visually presented condition with the exception of the presence of the shapes during E's trials explanation and the child's performance on the five trials with the five shapes. In the spatial cues verbally presented condition, E was again seated across from the child. However, the table was empty and E delivered only a verbal description of the shapes. E said, "I'm going to describe a shape to you" at the beginning of each geometric form description which required 10 seconds to relate. Exposure time to the form's characteristics was thus held constant since children in the spatial cues visually presented condition viewed the solid form 10 seconds before beginning the trials.

E began by depicting each of the geometric forms manipulations required by the trials but used only a verbal description of the cube for each trial. E then proceeded through all trials on each shape, substituting a verbal description for the visual display. All other language and reinforcement delivery remained identical. E made the same directional slashes and gestures with his/her hands in front of the child to indicate the nature and direction of the manipulations to be taken.

Subjects in both the verbal and visual presentation of spatial cues conditions were provided with a label (i.e., house, cone, etc.) for each of the geometric forms. In piloting, the verbal descriptions of some of the objects did not seem to be meaningful to the children until a label was also attached as a cue for the description that followed. Since it

was the extended description which had been designed to match the actual geometric forms, it was necessary to control for any possible effects of the label, by including the label in both conditions.

6. Post-Experimental Conditions Procedure

After responding to all trials for the five shapes, subjects in both conditions were given praise which implied a successful performance. The next step in the experimental procedure consisted of a shape identification segment to ascertain that the verbal descriptions contained meaningful information about the forms' characteristics. Subjects in both groups were given the shape descriptions used in the verbal condition and asked to point to what had been described. Eight shapes consisting of the six shapes previously used plus a triangular solid and trapezoid, as well as five perspective drawings of the five experimental shapes, were randomly arrayed on the table. However, the drawings were always placed flat in a random line in front of the actual forms to avoid their being blocked by the three-dimensional objects. The subjects turned their backs to the table while the objects were being arranged and rearranged after each response and while the descriptions were being given. After they made their first selection, subjects were asked if there was anything else that fit the description. For each description there was both a two-dimensional and three-dimensional correct response. The descriptions were presented in the same randomly derived order for all subjects. The order of presentation was as follows: Cone, rectangle, pyramid, cylinder, house.

C. Results

1. Correct Response Analysis on Transformation Tasks

The data were analyzed by means of a 2(grade)x2(sex)x2(premeasure type)x2(spatial cues presentation)x5(operation type) analysis of variance with repeated measures on the last factor. Significant main effects were found for grade ($F\ 1, 144 = 63.64$, $p < .001$), for spatial cues presentation ($F\ 1,144 = 31.55$, $p < .001$), and for operation ($F\ 1,576 = 69.17$, $p < .001$). Significant first-order interactions were detected for type of operation by grade ($F\ 4,576 = 3.47$, $p < .05$), operation type by spatial cues presentation ($F\ 1,576 = 4.38$, $p < .01$), type of operation by sex ($F\ 1,576 = 2.76$, $p < .05$), and pre-measure by spatial cues presentation ($F\ 1,144 = 4.95$, $p < .05$). No other significant interaction effects were

detected. The ANOVA is summarized in Table 20.

The main effects for grade and spatial cues presentation were as predicted with fifth graders performing better than second graders and with the visual presentation of spatial cues proving more facilitative than the verbal presentation. A post hoc analyses of the operation trials effect revealed that all of the pairwise comparisons on operations were statistically significant with the exception of the contrast between the double horizontal section and the horizontal section.

The Neuman-Keuls procedure was used to further analyze the interaction effects. Investigation of the type of operation by grade interaction depicted in Figure 31 revealed that all the contrasts were significant at the $p < .01$ level. The performance discrepancy between the two grades of the greatest magnitude comes from the horizontal section operations with the least differentiation being made by the unfolding operations.

Figure 32 represents the interaction of Operation Type x Spatial Cues presentation. This graph indicates that while the group which received a visual presentation of the spatial cues, differed from the verbal presentation group on the horizontal section, vertical section, double horizontal section and bottom operations, there was no difference between the two presentation conditions for the unfolding operation.

The Type of Operation by Sex interaction represented in Figure 33 reveals very few differences between either the pattern or magnitude of variance between males and females. Further analysis and interpretation of this interaction thus appears unwarranted.

The Pre-Measure by Spatial Cues presentation depicted in Figure 34 has one significant contrast. The labeling pre-measure had a significant effect only between presentation conditions. Those subjects who went on into the Visual Condition performed significantly better than subjects who proceeded in the Verbal Condition.

2. Analysis of Shape Identification

Data from the shape identification portion of the experiment were analyzed using a 2(grade)x2(spatial cues presentation)x2(dimensionality of response) analysis of variance with repeated measures on the last factor. Significant main effects were found for grade and dimensionality

Table 20. Transformation Task: Summary of Analysis of Variance

Source	df	MS	F	p
Between Groups				
Grade	1	243.10	63.64	p<.001
Sex	1	.78	.20	n.s.
Pre-measure	1	.55	.14	n.s.
Spatial cues presentation condition	1	120.90	31.65	p<.001
Grade x sex	1	.01	.003	n.s.
Grade x pre-measure	1	9.46	2.48	n.s.
Grade x condition	1	.55	.14	n.s.
Sex x pre-measure	1	.45	.12	n.s.
Sex x condition	1	.78	.20	n.s.
Pre-measure x condition	1	18.91	4.95	p<.05
Grade x sex x pre-measure	1	2.53	.66	n.s.
Grade x sex x condition	1	.10	.03	n.s.
Grade x pre-measure x condition	1	.03	.01	n.s.
Sex x pre-measure x condition	1	.01	.003	n.s.
Grade x sex x pre-measure x condition	1	2.76	.72	n.s.
Error	144	3.82		
Within Groups				
Operation	4	73.32	69.17	p<.001
Grade x operation	4	3.68	3.47	p<.05
Sex x operation	4	2.92	.276	p<.05

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Table 20, Continued

Source	df	MS	F	p
Pre-measure x operation	4	1.58	1.49	n.s.
Condition x operation	4	4.64	4.38	p<.01
Grade x sex x operation	4	.66	.62	n.s.
Grade x pre-measure x operation	4	1.07	1.01	n.s.
Sex x pre-measure x operation	4	1.32	1.26	n.s.
Grade x condition x operation	4	1.67	1.59	n.s.
Sex x condition x operation	4	1.52	1.45	n.s.
Pre-measure x condition x operation	4	.91	.86	n.s.
Grade x sex x pre-measure x operation	4	.51	.48	n.s.
Grade x sex x condition x operation	4	.75	.71	n.s.
Grade x pre-measure x condition x operation	4	.14	.13	n.s.
Sex x pre-measure x condition x operation	4	1.84	1.74	n.s.
Grade x sex x pre-measure x condition x operation	4	1.90	1.81	n.s.
Error	576	1.06		

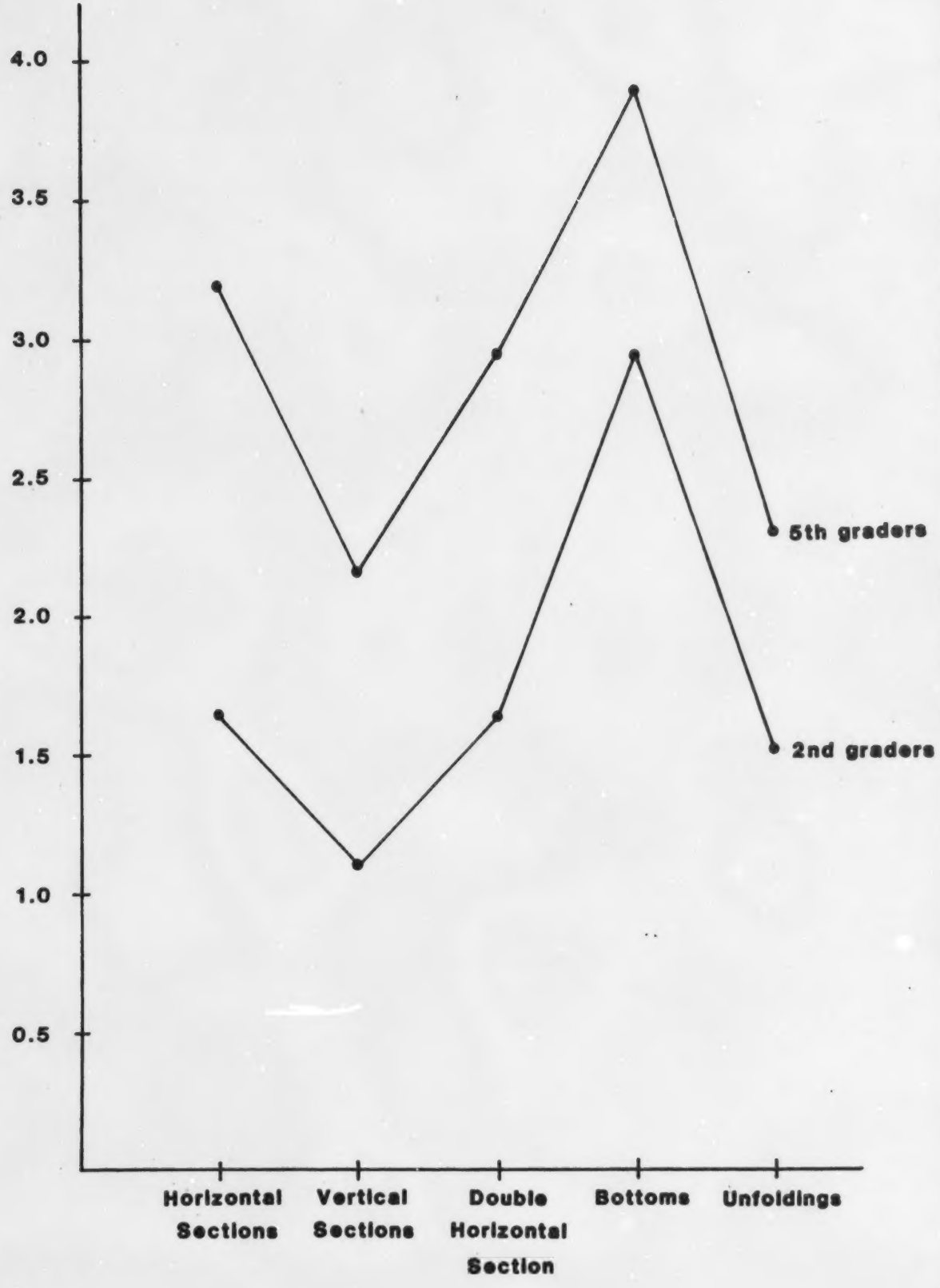

Figure 31: Type of operation by grade. Study 8 data

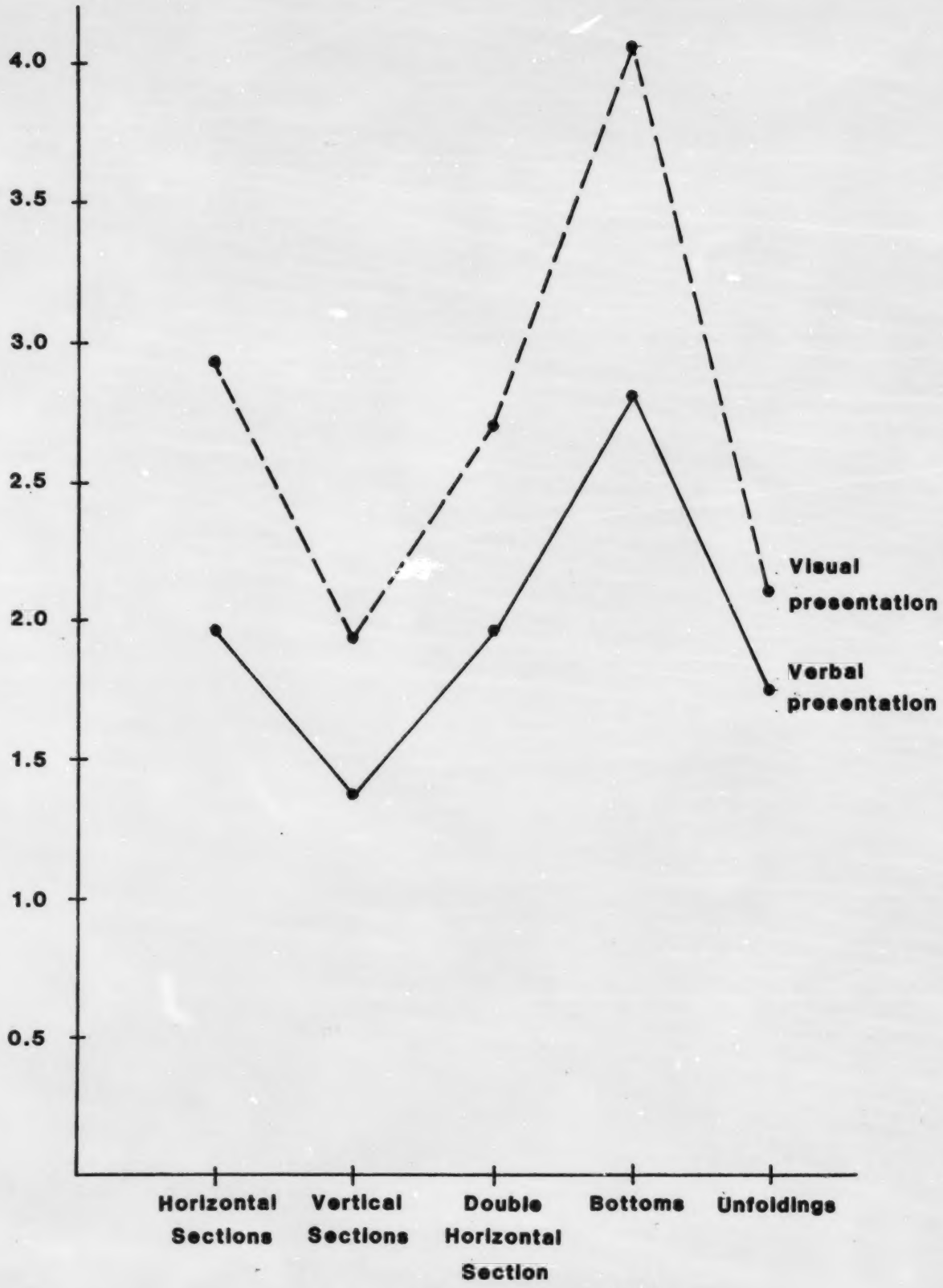

Figure 32: Type of operation by spatial cues presentation means. Study 6 data

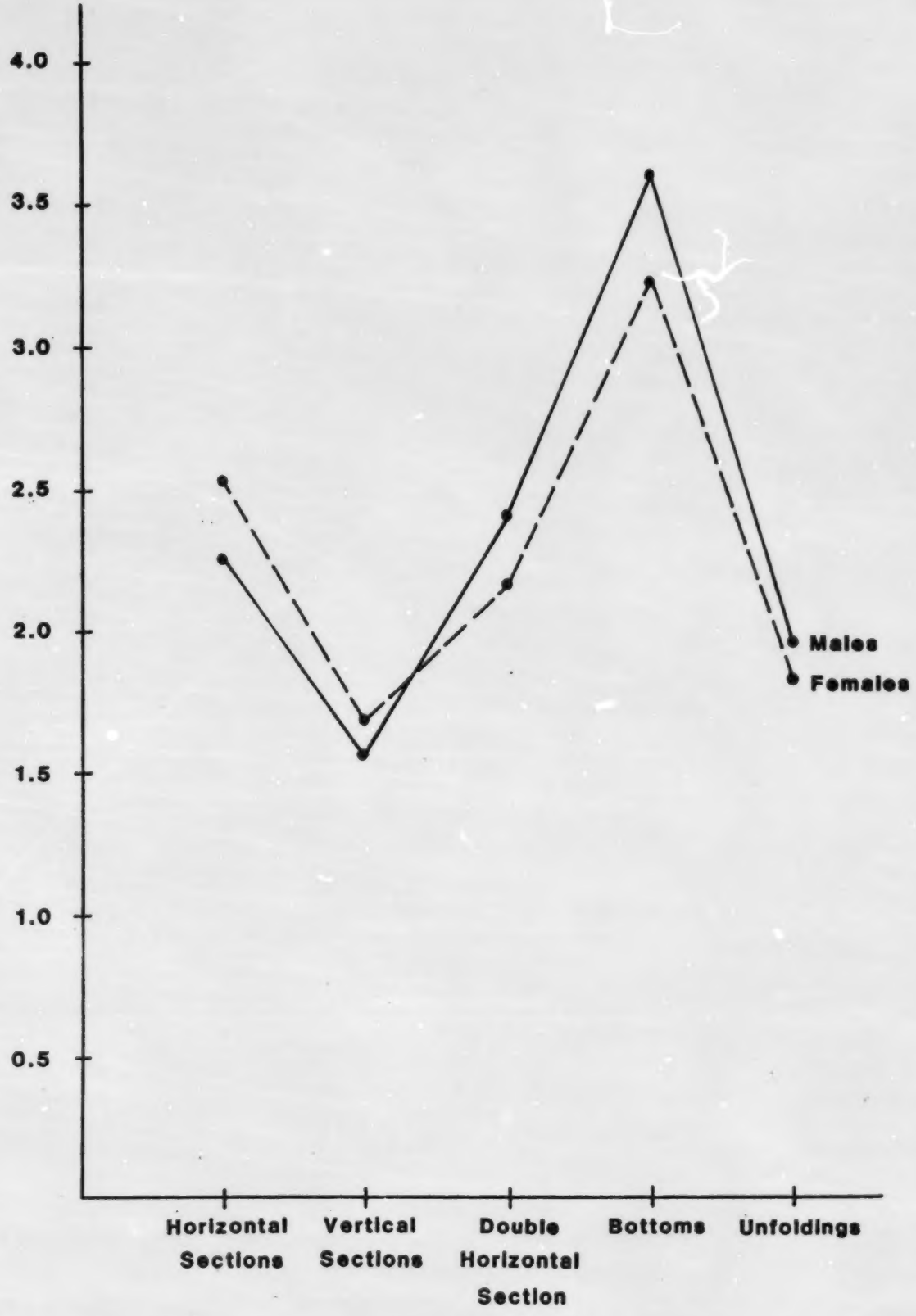

Figure 33: Correct response means for type of operation by sex. Study 8 data

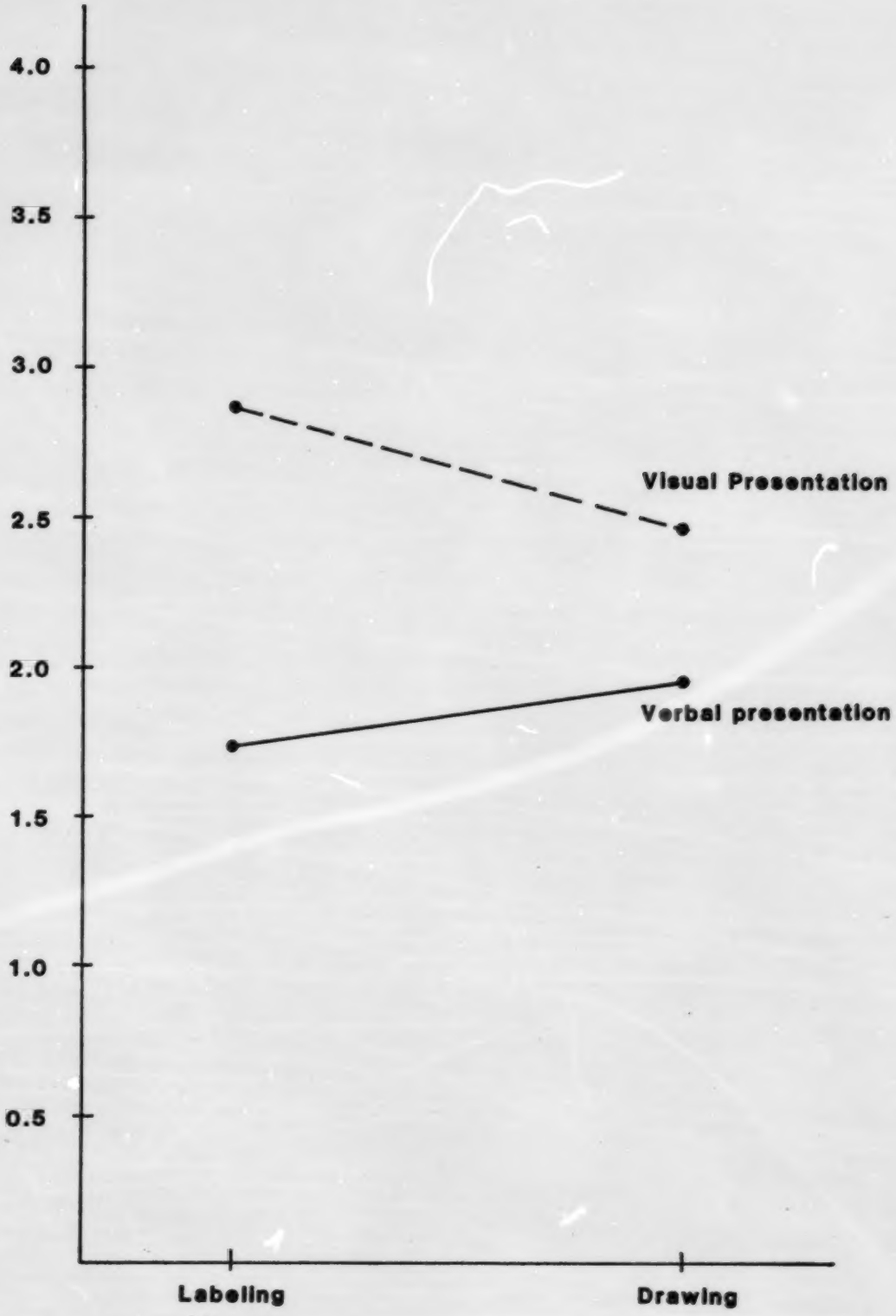

Figure 34: Correct response means for premeasure type by spatial cues presentation. Study 8 data

with fifth graders outperforming second graders and with three-dimensional forms selected more frequently than two-dimensional perspective representations of those forms.

Percentages of correct responses to each shape across spatial cues presentation conditions were also tallied. Responses were considered correct if an appropriate selection was either two-dimensional or three-dimensional. A score of 1 was assigned to a subject's response to each shape if one or both of the possible responses were correct. For each shape in each condition there was a total possible score of 20 correct responses. The range and percentage of correct responses to each shape in the two presentation conditions is reported in Table 21.

3. Analysis of Errors on Transformational Tasks

An analysis of the errors made during the transformation tasks was conducted employing the Likelihood-Ratio Chi-Square Statistic (Kulback, Kupperman, & Ku, 1962). Similar to the Pearson Chi-Square Statistic, the Likelihood-Ratio Chi-Square Statistic allows one to determine whether two variables are independent. However, the Likelihood-Ratio Statistic also provides information about the source of non-independence in a RxC contingency table, allowing one to partiion the contingency table into as many independent components as the table has degrees of freedom. Each subtable may then be tested with the Likelihood-Ratio Statistic to isolate significant sources of non-independence (Everitt, 1977).

A number of the comparisons between the type of operations and foil type yielded significant over-all Likelihood-Ratio Chi-Squares. Comparison of the horizontal and vertical sections operations depicted in Table 22 found they were not independent of one another by foil type (X^2_L = 67.14, df 4, $p < .001$). Neither were the comparisons of the vertical section and unfolding operations by foil type (X^2_L = 122.56, df 4, $p < .001$) shown in Table 23 and the horizontal section and unfolding operations by foil type (X^2_L = 24.36, df 4, $p < .001$) displayed in Table 24.

By collapsing column 2 of Table 22 with column 5 and Table 22's column 3 with column 4, a more parsimonious model (Table 25) was derived

$$(X^2_{L\ \text{Model 1}} - X^2_{L\ \text{Model 2}} = 7.53, \text{ df } 3, p < .05)$$

Table 21

Range and Percentage of Correct Shape Identification Responses by Shape and Spatial Cues Presentation Condition

	Visual		Verbal	
Shape	Range	Percentage	Range	Percentage
Cone	19-20	98	19-20	99
Rectangular solid	15-19	84	15-19	81
Pyramid	17-20	94	15-19	88
Cylinder	19-20	98	17-20	93
House	18-20	98	18-20	98

Table 22

Operation by Foil Type (Model 1)[a]

Operation	Foil Type 1	2	3	4	5
Horizontal section	74 (126)	103 (84)	25 (16)	107 (105)	108 (86)
Vertical section	214 (162)	88 (107)	11 (20)	132 (134)	88 (110)

Table 23

Operation by Foil Type (Model 1)[a]

Operation	Foil Type 1	2	3	4	5
Vertical section	214 (139)	88 (98)	11 (12)	132 (158)	87 (125)
Unfolding	56 (131)	101 (92)	13 (12)	174 (148)	155 (117)

Table 24

Operation by Foil Type (Model 1)[a]

Operation	Foil Type 1	2	3	4	5
Horizontal section	74 (59)	103 (93)	25 (17)	107 (128)	108 (120)
Unfolding	56 (71)	101 (111)	13 (21)	174 (153)	155 (143)

[a]Numbers in parentheses indicate the expected frequencies.

for the horizontal and vertical sections operations comparison (Fienberg, 1977). Table 26 depicts a simpler model for the vertical section and unfolding operations, derived by collapsing together columns 2, 3, 4, and 5 of Table 23.

$$(X^2_{L_{\text{Model 1}}} - X^2_{L_{\text{Model 2}}} = 5.80, \text{ df } 3, p < .05).$$

The original model for the horizontal sections versus unfoldings operations is more parsimoniously described by a second model (Table 27) which collapsed columns 2 and 3 of Table 24.

$$(X^2_{L_{\text{Model 1}}} - X^2_{L_{\text{Model 2}}} = 3.40, \text{ df } 1, p < .05).$$

Comparisons were also made of the foil type by sex, grade level and spatial cues presentation for the unfoldings, horizontal sections and vertical sections operations. The variables of sex and foil type were independent of one another for all three operations. Spatial cue presentations and foil type also proved independent for the horizontal and vertical sections operations. However, Table 28 shows that a comparison of the spatial cues presentation condition by foil type was significant for the unfolding operation

$$(X^2_L = 17.60, \text{ df } 4, p < .01),$$

indicating a lack of independence for these variables in this particular operation. A more parsimonious model outlined in Table 29

$$(X^2_{L_{\text{Model 1}}} - X^2_{L_{\text{Model 2}}} = 2.08, \text{ df } 3, p < .05)$$

collapses columns 1, 2, 3 and 5 of Table 28.

Table 30 indicates that grade level was not independent of foil type for the unfolding operation

$$(X^2_L = 13.08, \text{ df } 4, p < .02).$$

A simpler model for this comparison is depicted in Table 31 and collapses together columns 1, 2, 3, and 4

$$(X^2_{L_{\text{Model 1}}} - X^2_{L_{\text{Model 2}}} = 4.57, \text{ df } 3, p < .05).$$

Table 25

Operation by Foil Type (Model 2)[a]

Operation	Foil Type 1	2 & 5	3 & 4
Horizontal section	74 (126)	211 (170)	132 (121)
Vertical section	214 (162)	176 (217)	143 (154)

Table 26

Operation by Foil Type (Model 2)[a]

Operation	Foil Type 1	2, 3, 4, & 5
Vertical section	214 (139)	318 (393)
Unfolding	56 (131)	443 (369)

Table 27

Operation by Foil Type (Model 2)[a]

Operation	Foil Type 1	2 & 3	4	5
Horizontal section	74 (59)	128 (110)	107 (128)	108 (120)
Unfolding	56 (71)	114 (132)	174 (153)	155 (143)

[a]Numbers in parentheses indicate the expected frequencies.

Table 28

Spatial Cues Presentation by Foil Type Analysis of Unfolding Operation (Model 1)[a]

Presentation	Foil Type 1	2	3	4	5
Verbal	36 (29.63)	61 (53.43)	9 (6.88)	71 (92.06)	87 (82.00)
Visual	20 (26.37)	40 (47.57)	4 (6.12)	103 (81.94)	68 (73.00)

[a]Numbers in parentheses indicate the expected frequencies.

Table 29

Spatial Cues Presentation by Foil Type Analysis of Unfolding Operation (Model 2)[a]

Presentation	Foil Type 1, 2, 3, & 5	4
Verbal	193 (171.94)	71 (92.06)
Visual	132 (153.06)	103 (81.94)

[a]Numbers in parentheses indicate the expected frequencies.

Table 30

Grade Level by Foil Type Analysis of Unfolding Operation (Model 1)[a]

Grade	Foil Type 1	2	3	4	5
2nd	35 (31.42)	68 (56.67)	9 (7.29)	96 (97.64)	72 (86.97)
5th	21 (24.58)	33 (44.33)	4 (5.71)	78 (76.36)	83 (68.03)

[a]Numbers in parentheses indicate the expected frequencies.

Table 31

Grade Level by Foil Type Analysis of Unfolding Operation (Model 2)[a]

Grade	Foil Type 1, 2, 3, & 4	5
2nd	208 (193)	72 (87)
5th	136 (151)	83 (68)

[a]Numbers in parentheses indicate expected frequencies.

Error type and grade level (Table 32) were also non-independent variables for the vertical section operation

$$(X^2_L = 20.32, \text{ df } 4, p < .001).$$

Columns 1, 2, 3, and 5 of Table 32 are collapsed together in Table 33 resulting in a more parsimonious model than the original

$$(X^2_{L_{\text{Model 1}}} - X^2_{L_{\text{Model 2}}} = 3.29, \text{ df } 3, p < .05).$$ Finally, the comparison of grade level by foil type for the horizontal sections operation was also significant

$$(X^2_L = 30.94, \text{ df } 4, p < .001)$$

as delineated in Table 34. This table was simplified by the collapsing together of columns 2, 3, and 4. The less complex model for this final comparison is represented in Table 35

$$(X^2_{L_{\text{Model 1}}} - X^2_{L_{\text{Model 2}}} = .02, \text{ df } 2, p < .05).$$

D. Discussion

This research addresses a number of issues important to educators, developmentalists and cognitive psychologists. In the developmental realm, the results of the analysis of variance have demonstrated, unsurprisingly, that fifth graders make more correct responses than do second graders. Furthermore, contrary to the kinds of qualitative differences in response patterns between the two grades which one would predict from a Piagetian perspective, the pattern of responding across the five operation types is essentially equivalent. This similarity in response patterns could lead one to hypothesize that the subjects of the two grade levels differ only in proficiency, not in their manner of problem solving.

However, although the overall pattern of correct responses to the five types of operations is similar, the errors analysis does reveal some differences in the type of errors made. For example, when the errors made on the unfolding operations were analyzed by means of a Likelihood-Ratio Chi-Square statistic, it was found that foil number 5 was independdent of the combination of the other four foils. Fifth graders made fewer of the other four foil types than would be expected but concomitantly made more choices of foil 5. Foil 5 contained all of the characteristics of the

Table 32

Grade Level and Foil Type Analysis of Vertical Section Operations (Model 1)[a]

Grade	Foil Type 1	2	3	4	5
2nd	101 (113.9)	44 (50.12)	8 (6.27)	73 (55.25)	77 (77.46)
5th	99 (86.09)	44 (37.9)	3 (4.73)	24 (41.75)	59 (58.54)

[a]Numbers in parentheses indicate expected frequencies.

Table 33

Grade Level and Foil Type Analysis of Vertical Section Operations (Model 2)[a]

Grade	Foil Type 1, 2, 3, & 5	4
2nd	230 (247.75)	73 (55.25)
5th	205 (187.25)	24 (41.75)

[a]Numbers in parentheses indicate expected frequencies.

Table 34

Grade Level and Foil Type Analysis of the Horizontal Section Operations (Model 1)[a]

Grade	Foil Type 1	2	3	4	5
2nd	32 (48)	64 (66)	16 (16)	68 (69)	89 (70)
5th	42 (26)	39 (37)	9 (9)	39 (38)	19 (38)

[a]Numbers in parentheses indicate expected frequencies.

Table 35

Grade Level and Foil Type Analysis of the Horizontal Section Operations (Model 2)[a]

Grade	Foil Type 1	2, 3, & 4	5
2nd	32 (48)	148 (151)	89 (70)
5th	42 (26)	87 (84)	19 (38)

[a]Numbers in parentheses indicate expected frequencies.

correct response with the exception of slight but inaccurate proportions changes which could be viewed as a minor error. Fifth graders were deceived by this slight error more often than by the more obvious error types. Second graders on the other hand tended to make errors reflective of operating upon the wrong image, failure to account for all sides of the form, or a totally inaccurate response.

Similarly, the older children made more of the error of the smallest magnitude when performing the horizontal sections operation. Second graders tended to select a two-dimensional representation of the form which had not been operated upon, foil 5, thus indicating a failure to understand the requested operation. A larger number of fifth graders than would be predicted chose foil 1 which depicted the result of performing the correct operation upon a two-dimensional perspective representation of the object. Again, for this operation, foil 1 seemed to be a relatively minor distortion.

Finally, for the vertical sections operations, foil 4 and a combination of the other four foils, differentiate the second and fifth graders. More second graders and fewer fifth graders than would be expected chose foil 4, which represented a horizontal section instead of the requested vertical section. As in their performance on the horizontal section operations, this indicated a failure on the part of the second graders to perform the necessary operation. Fifth graders, on the other hand, chose more than predicted of the other foil types than did the second graders.

The errors analysis reveals that despite the similarity between the second and fifth graders on correct response patterns, the two grade levels do differ in their error patterns. When fifth graders err they generally do not choose many foils which represent a failure to understand the required operation. Rather, their error choices tend to be those of more subtle deviation. Second graders, however, are more attracted to foils which contain errors on a number of the characteristics of the correct response. They were not deceived by the error in proportionality for the unfolding operation because they generally were incapable of attaining that high level of sophistication. For the horizontal and vertical sections operations, the second graders were attracted to operational errors. Vertical section operations most frequently resulted in a confusion of the

vertical and horizontal.

Other researchers have discovered that horizontality seems to be a concept which arises earlier in the developmental sequence and is more easily grasped than verticality (Abravanel & Gingold, 1977; Berman, 1976; Berman & Cunningham, 1977; Berman, Cunningham, & Harkulich, 1977; Bryant, 1969; Liben, 1978b; McGillicuddy-DeLisi, DeLisi, & Youniss, 1978). Thus, while children may confuse a vertical section with a horizontal section, one would not expect the reverse to be true, a fact borne out by the present data. An examination of performance across operation type suggests a hierarchical arrangement of operation type in the following ranking: (1) bottoms, (2) horizontal and double horizontal sections, (3) unfolding, and (4) vertical section operations in order from easiest to most difficult.

Finally, the comparison of the choice of foil types between vertical and horizontal sections operations and across grades revealed that subjects made different errors in responding to the two tasks. Errors made during vertical section taking tended to be errors resulting from operating on a two-dimensional perspective image. For the horizontal section operations subjects erred by operating on a two-dimensional non-perspective image or by failing to operate at all and simply selecting the whole image foil.

Regarding the issue of differential performance between males and females, this research has failed to find a main effect for sex. While there was a significant first-order interaction for type of operation performed by sex, the differences between males and females were so small and their overall response patterns so similar, that an interpretation of this interaction would seem unwise. Moreover, the errors analysis failed to yield any evidence of differential performance.

This study also provides some eveidence that different formats of inputting data will engender differential results. An assumption made in designing this research was that a mediating representation would be necessary to both store information about the form's characteristics before transformation, and to regenerate the original representation, even after some of the transformational operations had been performed. It was further

assumed that this mediating representation would assume the characteristics of spatial imagery in the manner described by Kosslyn (1978).

Kosslyn (1978) has suggested that a spatial representation is generated from a combination of perceptual information, or memories of the actual appearance of an array, with the non-perceptual information that would be contained in verbal descriptions or abstract propositions. This spatial representation is an image, a surface representation derived from deep representations stored in long-term memory in both a perceptual and propositional format.

Kosslyn's position has been vigorously attacked (Anderson, 1978; Pylyshyn, 1973, 1979) by propositional theorists who claim that all information is stored in an abstract format, devoid of any perceptual characteristics. While this research does not purport to challenge either represenatational format, some of the findings may have relevance to this controversial issue.

The correct response analysis of the spatial cues presentation conditions demonstrates that the visual presentation of spatial cues was more facilitative of performing the various operations than was the verbal persentation. While it might be logical to suggest that the visual presentation contained more relevant information about the forms than did the verbal presentation, results of the shape identification task question the validity of such a conclusion.

The shape identification segment of the experimental session was conducted to ascertain whether the descriptions given to the subjects in the verbal presentation of spatial cues contained meaningful information about the forms' characteristics which was equivalent to the visually presented information. The percentages of correct identification responses to each description between presentation groups were essentially equal for all the forms. Furthermore, an analysis of variance of this shape identification task found no significant differences between the two presentation conditions. The observed operational performance difference favoring the visual presentation condition should not be attributed solely to the communication of less meaningful information in the verbal presentation of spatial cues.

A more probable interpretation is that both presentation conditions communicated essentially equivalent information but provided differential access to spatial imagery. Kosslyn (1978) and Finke and Kosslyn (1980) have suggested that perception and mental imagery share common processes. Furthermore, if the image does have many of the spatial characteristics of a visual percept (Kosslyn, 1978) then a visual presentation of spatial cues should provide a more direct access to spatial imagery than would a verbal presentation. Assuming a closer match between a visual percept and imagery and perception of verbal information and imagery, one could hypothesize a simple model depicted in Figure 35, based on Kosslyn's ideas for the encoding and representing of information. Thus, a verbal presentation of spatial cues should yield a more indirect access to spatial imagery than would a visual presentation, since it entails an extra processing step. The visual presentation model presents information which closely matches that requisite for an image, since the perceptual information is encoded spatially. That is not true of the verbal presentation in which the perceptual information is encoded in a distinct verbal code which must then be transformed prior to the generation of a spatial image.

Similar models based on propositional theory fail to differentiate between the two presentation conditions, suggesting that spatial imagery was the mediator in these tasks. Corroboration for this hypothesis is drawn from the consistently similar patterns of responding across operation types between the two presentation conditions (Figure 36). Although the unfolding operation fails to differentiate between the two conditions, the errors analysis pinpoints this variance as lying in differential responses across conditions to foil 4 and to a combination of foils 1, 2, 3, and 5. Visual presentation subjects chose foil 4 more than would be expected while choosing fewer of the other types of errors. However, since foil 4 was an accurate representation which erred by deleting one side of the form, this error can be viewed as careless and not necessarily deleterious to the differential access hypothesis.

An alternate explanatory hypothesis for differential operational performance across presentation conditions is that the visual presentation may have been more facilitative of the formation of an accurate three-dimensional image than was the verbal condition. However, the absence

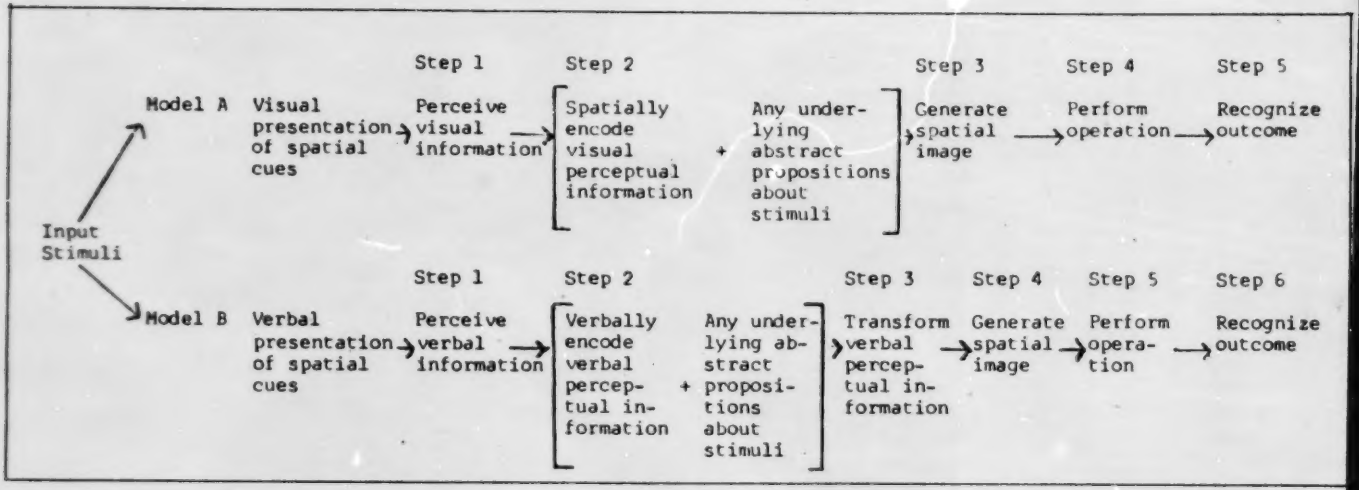

Figure 35. Information-Processing Models Based on Imagery Theory

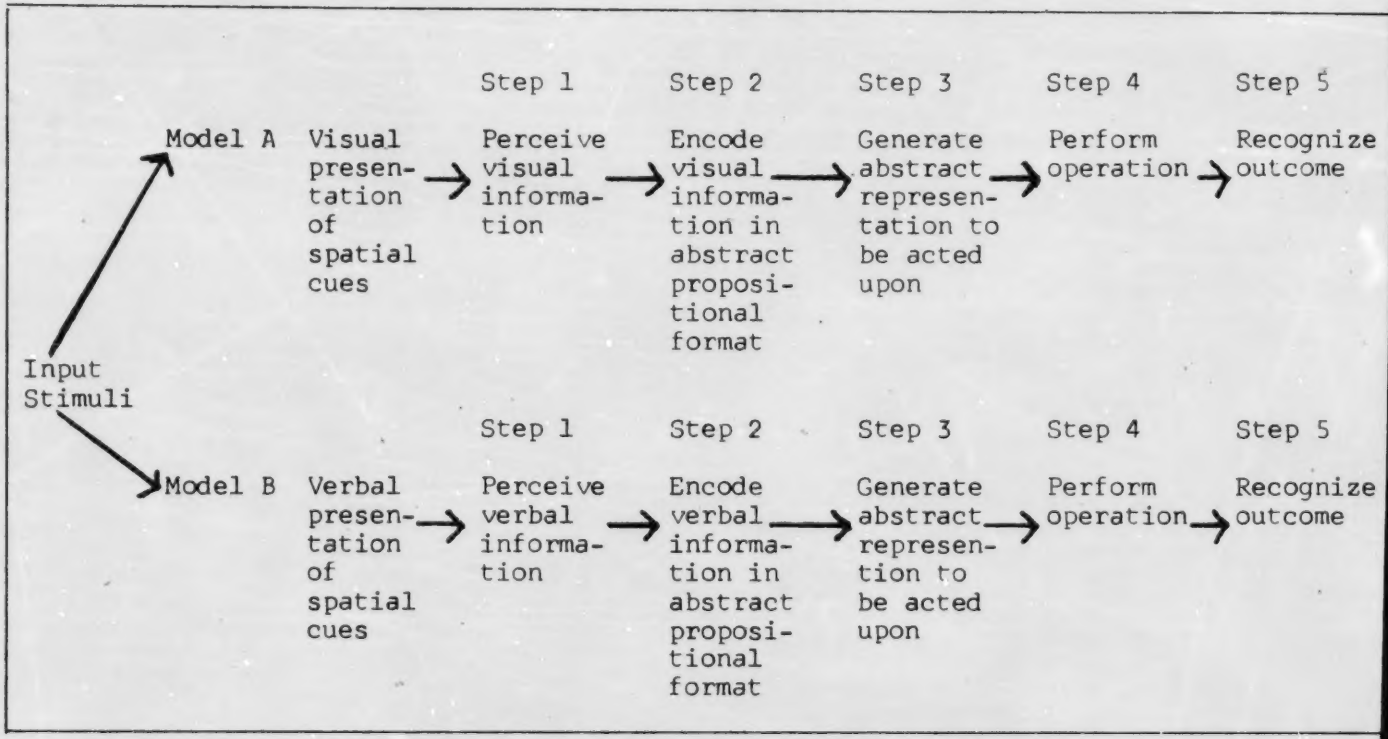

Figure 36. Information-Processing Models Based on Propositional Theory

of a significant difference between those foils based on correctly operating upon inappropriate imagery (foils 1 and 2) and foils based on employing the wrong operations on correct imagery (foils 4 and 5) suggests that imagery differences did not account for much variance. The assumed absence of imagery differences is further supported by the lack of interaction between presentation condition and response dimensionality on the shape identification task. The greater frequency of three-dimensional responses, despite the fact that the two-dimensional depictions were closer to subjects when they indicated their choices, suggests that subjects' imagery may have tended to be more three-dimensional than two-dimensional. Such findings concur with Metzler and Shepard's (1974) observation that the internal representation of a stimulus item shares more characteristics with the actual three-dimensional object than with its two-dimensional perspective depiction.

This research also provides support for employing an errors analysis to detect subtle differences in responding. The errors analysis indicated that the type of operation being performed would yield very different results in those subjects who responded incorrectly. Such subtle differences should prove useful in designing further research.

In summary, the results of this investigation suggest that there are differences in processing information presented verbally versus information presented visually. These differences may result from the more direct access to a spatial image mediator provided by a visual presentation of cues. Subsequent research should further explore the relationship between verbal and visual presentations and spatial performances. Investigations of this type would be beneficial to curriculum development and classroom instruction.

VI. General Discussion

A. Spatial Abilities and Sex Differences

In this report the results of eight studies in spatial competencies have been summarized, the developmental implications for future research discussed, and indications for future research suggested. As a set, the studies cover a wide age range and were designed to tap two of the factors, spatial orientation and visualization, identified by Lohman (1979a). The tasks employed vary from very simple perceptual tasks like those in the

first study, to the sophisticated transformational ones in the last. Therefore, taken as a set, the scope of the research is comprehensive. The major developmental trend across studies, indexed by the age factor, appears to be a fairly linear quantitative growth pattern common to each. In all studies, age was found to be a statistically significant factor. The other attribute variable incorporated across all studies was sex. Here the result is also uniformly consistent: No significant sex differences were detected. However, though both findings are unequivocal in respect to their consistency, the interpretation and meaning of the two findings differ.

A replicated significant effect comments on the state of reality and becomes solid evidence for the rejection of a null hypothesis. However, a replicated non-significant effect cannot necessarily be used in support of the null. The findings of no statistical difference on the sex variable in these studies and in many others like them which examine spatial skills (e.g., Sherman, 1978) presents just such a problem since it is empirically impossible to "prove" the null hypothesis of no difference on the basis of sex. In order to address this substantive concern, therefore, one probably requires a difference approach.

Failure to reject the null will occur whenever: (1) the null does in fact accurately reflect the state of reality, (2) the estimate of within variance, that is error, is relatively large, or when (3) for other reasons the statistical analysis lacks sufficient power. Most of the time selecting the one reason out of the many to explain lack of significant difference findings is problematic. In these eight studies neither insufficient power nor excessive error appear as highly plausible explanations since estimates of error variance were uniformly small and highly reliable instruments were typical. Using this information to evaluate the null still remains problematic.

One procedure that can be pursued which places the magnitude of an obtained group difference in context is the computation and meta-analysis of effect sizes (Glass, 1978). We chose this procedure as one means of evaluating obtained though statistically insignificant differences on the sex factor across our studies. In computing effect sizes, lack of

precision and/or extent of individual differences is taken into account since the mean contrast is compared with an estimate of variability. The resultant effect size is expressed in standard deviation units. Effect sizes, since they are thus converted to an equivalent scale, can be directly compared, averaged, etc., like any z score.

We had several questions we wished to address which could not be answered on the basis of the previous statistical data. For example, would the sexes differ depending on the type of task utilized? Would they differ in accord with different factors which the tasks were designed to tap? Those are questions which cannot be addressed through within-study analyses since they target across-study issues. Computation and comparison of effect sizes might at least partly illuminate those queries.

When the project studies were originally proposed, one of our target concerns was whether or not children could cross dimensions in solving spatial tasks. Our earlier work suggested that dimensional demands could present a problem for young children (Horan & Rosser, 1979). Study 4 and study 7 included dimension-crossing as an experimental factor. However, we did not directly compare within dimension performance with across dimension performance in any one study. Summary data suggest that crossing dimensions was harder for children than operating in only one, but the age effects were parallel across the first four studies. Would the sex effect be similarly parallel? Again this is a question which might be addressed through the examination of effect sizes.

To address these cross-study issues, we performed the following operations: (1) effect sizes were computed for each study and are presented in Table 36 (2) effect sizes were averaged across possible factors and across all studies. These data are presented in Tables 37 and 38; (3) and finally effect sizes were compared for studies which did not require dimension-crossing with those which did. This information appears in Table 39. The averaging of effect sizes was conducted in two ways. In the first, effect sizes were simply averaged as z scores would be. In the second, each effect size was weighted by the number of subjects in the study from which the effect size came. Therefore, two tables represent each question addressed with both methods.

Table 36. Effect size across the studies for the sex factor

Study	Number of Subjects	Effect Size	
		Male Advantage	Female Advantage
1	60		.32
2	54		.29
3	54		.17
4	108	.11	
5	120		.92
6	60		.53
7	108	1.05	
8	160	.15	

Table 37. Effect size for the sex difference by the type of task: Unweighted analysis

Task	Studies Compressed	Effect Size
Low level visualization	1-4	.17 for females
Spatial orientation	5-7	.13 for females
Transformational visualization	8	.15 for males
General spatial skills	1-8	.12 for females

Table 38. Effect size for the sex difference by type of task; weighted analysis

Task	Studies Compressed	Effect Size
Low level visualization	1-4	.12 for females
Spatial orientation	5-7	.10 for females
Transormational visualization	8	.15 for males
General spatial skills	1-8	.05 for females

Table 39. Effect size for the sex difference by dimensionality requirements

Task	Unweighted Analysis	Weighted Analysis
Within dimension tasks (study 1,2,3,5,6)	.45 favoring females	.54 for females
Across dimension tasks (study 4,7,8)	.44 for males	.40 for males

Examination of these tables is intriguing. In terms of overall compression, the meta-analysis clearly suggests an absence of a sex effect since with whatever method used, average effect size is negligible. Similarly a compression on the basis of possible factor tapped reveals inconsequential effect sizes. Inspecting the data across studies in this fashion would perhaps lead one to strongly support, if not to accept, the null hypothesis.

However, a different conclusion is suggested when the issue of dimension crossing becomes the basis of compression. Whether a weighted or unweighted approach is used, females display the advantage when the task is within dimensions, and males display it when crossing dimensions. Effect sizes of .4 or greater do not seem as inconsequential as do the others. We further pursued this analysis by compartmentalizing the tasks. Studies 1, 2, 3, and 4 employ identical tasks. Studies 1, 2, and 3 stay within dimensions, and study 4 crosses dimensions, but all tap low-level visualizing with duplicate Euclidean displays. Therefore, the compressed effect sizes of 1, 2, and 3 were compared with that for study 4 and is presented in Table 40. Similarly, tasks tapping the spatial orientation factor (5, 6, and 7) were examined, and the same-dimension studies (5 and 6) contrasted with the across-dimension one (study 7). These data are also presented in Table 40.

Table 40

Dimensionality x Task Type Comparisons

of the Effect Size for Sex

Task	Studies	Unweighted	Weighted
Low level visualization			
Within dimension	1-3	.26 for females	.28 for females
Across dimensions	4	.11 for males	.11 for males
Spatial orientation			
Within dimensions	5,6	.73 for females	.79 for females
Across dimensions	7	1.05 for males	1.05 for males

Tables 39 and 40 indicate that when dimensionality requirements change, not only do effect sizes change, but the ranking of the groups changes as

well. Females evidence superiority on the within-dimensionality tasks, males on the across-dimensionality ones. If mean contrasts are always made on the basis of the same order, i.e.,

$$\frac{\bar{X}_{\text{males}} - \bar{X}_{\text{females}}}{\text{S.D.}}$$

then female group advantages become negative z scores and male ones positive z scores. Employing these effect sizes, we can directly compare the change in ranking and distance between groups in standard deviation (SD) units. This difference for low-level visualization for the unweighted and weighted procedure is .37 SD and .39 SD, respectively. For spatial orientation it is 1.78 SD and 1.84 SD. These last quantities are not negligible. It would appear from the application of the meta-analysis technique that effect sizes for sex are most pronounced in the comparisons made on the basis of dimensionaltiy. Moreover, the most dramatic contrasts are on spatial orientation tasks. In fact, these differences are larger than many others reported for the spatial area (Sherman, 1978) and were found here with young children (4-8 years). Even more intriguing is that the subjects who participated in studies 5, 6, and 7 were the same children and yet the group means on the sex variable reverse ranking by almost 2 standard deviations when dimensionality transformations were required!

While the meta-analysis technique for examining this kind of data is explanatory since the number of studies considered here is not great, this approach is at least suggestive of future research hypotheses. For example, most summaries of sex differences studies attempt to evaluate and compress large numbers of studies which vary in terms of the tasks employed and the psychological processes tapped. There may not be a valid psychological reason for grouping these tasks as equivalent dependent measures when in all probability they tap different mental processes. Perhaps across-task evaluation, while revealing the general differences or lack of them on overall success in spatial abilities, masks other important but subtle processing differences.

The studies conducted here suggest two conclusions about sex differences in young children: (1) in terms of overall success, boys and girls do not appear to differ; (2) when it comes to specific processing abilities, they may. The latter cannot be concluded from these data alone. The

specific ability to transform a problem presented in one dimension to a solution in another could be one of those particular skills worth further examiantion. It is interesting to recall here the differences in dimensionality requirements between study 4 and study 7. In the first, the depth dimension was minimally used; 2- and 3-dimensional materials varied only in terms of some being flat strips and some being one inch high blocks. In study 7, however, the depth dimension was definitely included; blocks were placed standing on end as well as employing the vertical and horizontal axis of the plane. Furthermore, the sex differential is more pronounced in study 7.

Future research into sex differences probably needs to focus on hypotheses derived from concern with the specifics of processing rather than on global performance differences. For example, further meta-analysis on studies which employ a cross-dimensional response device could be attempted. In addition, studies should be designed which block on sex and compare performance when crossing dimensions and performing within dimensions. Interactions between dimensional characteristics and sex detected within such a study would directly evaluate the speculative conclusions from the meta-analysis conducted here.

B. Directions for Future Research

Much remains that we do not know about the processing of spatial information and the utilization of spatial imagery by children. Sophisticated research has been conducted with adults in the spatial area, and resultant theoretical debate about the nature of spatial processing has grown from this body of research. But much less information is forthcoming about synonomous processing in children. For purposes of psychological theory building, we need to know much more about how children encode, retain, transform, and retrieve spatial information, and moreover, how processing specifics change as a function of age. The area would seem to provide a rich source of hypotheses for future study. For example, it has been suggested that adults use different problem-solving techniques depending on the stimulus complexity of spatial tasks (Lohman, 1979a), is that also a possibility with children? How do children analyze and encode information presented spatially? How do they deal with interference in the visual channel when processing spatial information at the representational

level? What is the form of children's internal representations for spatial and visual information? These questions have been and are currently being examined with adults, but not with children.

With the exception of Piaget and Inhelder's work (1956, 1971) and the empirical investigations spurred by the resultant conclusions the Genevans offer, comprehensive analysis of spatial information-processing in children appears to be lacking. Processing models would be welcomed in this area to further direct our research. In addition, studies which examine how children acquire spatial competence, examining the mechanisms of spatial learning, are needed as well. We need to know how socialization practice influences spatial skills development; until we do, we can say very little about the origins of spatial competence, nor can we pursue curriculum innovation to foster them.

C. Implications for Education

1. From Research to Practice

At one point during the year's research activities, the principal investigator received a copy of an article located by one of her students. The article (which shall remain unidentified) was directed at science and mathematics teachers at the elementary level. Basically, the major premises of the discussion were (1) that the topological to Euclidean developmental sequence proposed by Piaget and Inhelder (1956) was the way children acquire geometric concepts, (2) that young children could not display competence in Euclidean knowledge nor be expected to learn Euclidean concepts, and (3) that formal instruction should match this natural sequence. While these conclusions were presented as fact, none of the current controversy or debate about the proposed sequence was included, nor was question ever raised that such controversy might exist. The results of the studies presented here, as well as empirical work by others, would not support blanket statements of this type.

While it is important and socially relevant to generalize findings from basic research to practice in educational settings, it seems unwise to be premature in formulating direct recommendations. Our current understanding of children's abilities to learn and process spatial and Euclidean information is still somewhat nebulous and requires refinement before

direct curriculum innovations should be pursued. The psychological literature is suggestive; it is not yet definitive.

2. Spatial Literacy

In formal educational settings, information is transmitted to learners by means of language, often in a prose form, and through the presentation of visual displays. No one assumes that children will automatically extract and comprehend messages presented in a prose format. In fact in recent years how children do learn from prose has become a popular educational research topic generating prolific investigations. An equivalent amount of interest and accompanying study apparently does not exist with respect to learning through a visual/spatial media. It is doubtful that children automatically and intuitively extract and comprehend messages presented in a visual format; how the process does occur requires research attention. Since visual presentation is used in the schools and is incorporated in instructional materials, it behooves us to learn more about visual processing in children.

The issue of dimensionality and the ability of young children to cross dimensions is a case in point. Much of the visual information presented to children assumes a two-dimensional format in textbook materials, worksheets, charts, etc., even when depicting three-dimensional phenomenon. Children may not automatically perceive the match across dimensional depictions; our work at least suggests that cross-dimensionality tasks are difficult ones for them to perform. Moreover, little girls may experience greater difficulty with dimension-crossing than little boys. If important concepts and mathematical principles are presented in this fashion, students less spatially capable may fail to learn the conceptual material. Perhaps direct instruction in spatial "literacy" needs to be examined, or investigations undertaken which directly compare visual and other modes as facilitative instructional procedures.

A child's environment is a spatial one as well as a verbal one. Spatial analogues conveying information are pervasive. Mathematics is a spatial as well as a quantitative realm. And many specific competencies require the ability to internally manipulate spatial information. In order to prepare learners in these necessary spatial abilities much more needs

to be known about age and sex differences, processing variables, and in particular the learning mechanisms which foster the growth and development of spatial competence.

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